5c

No. of printed pages: 2

Seat No. ______

[84 & A-53]

SARDAR PATEL UNIVERSITY

M.Sc. Mathematics, Semester - III

Thursday, 12th April, 2018

PS03CMTH02, Mathematical Methods - I

Maximum marks: 70

Time: 02:00 p.m. to 05:00 p.m.

Note: 1. Assume standard notations wherever applicable.
2. Figures to the right indicate full marks of the respective question.

Q-1 Choose the most appropriate option for each of the following questions: [8]

1. Fourier coefficient a_0 of the Fourier series of 2π-periodic function $f(x) = 1, -\pi < x \leq \pi$ is ____.
   (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
2. The Fourier series of a 2π periodic function f on $(-\pi, \pi]$ converges to $f(x)$ for every $x \in \mathbb{R}$ if $f(x)$ is ____.
   (a) $\frac{\sin x}{x}$ (b) $\chi_{\mathbb{Q}}$ (c) $\sin\left(\frac{1}{x}\right)$ (d) e^{x^2}
3. $F\left[9e^{-\frac{x^2}{2}}\right](s) =$ ____.
   (a) $\frac{1}{9}e^{-\frac{s^2}{2}}$ (b) $3e^{-\frac{s^2}{2}}$ (c) $9e^{-\frac{s^2}{2}}$ (d) $81e^{-\frac{s^2}{2}}$
4. If f is an odd integrable function, then $F[f] =$ ____.
   (a) $-F_s[f]$ (b) $F_s[f]$ (c) $-iF_s[f]$ (d) $-F_c[f]$
5. By convolution theorem, $L[f * g] =$ ____.
   (a) $L[f] * L[g]$ (b) $L[f]L[g]$ (c) $L[fg]$ (d) none of these
6. $L^{-1}\left[\frac{1}{(s-1)^n}\right](t) =$ ____.
   (a) $e^t\frac{t^{n+1}}{(n+1)!}$ (b) $e^t\frac{t^{n-1}}{(n-1)!}$ (c) $e^{-t}\frac{t^{n-1}}{(n-1)!}$ (d) $\frac{t^{n-1}}{(n-1)!}$
7. The derivative of the Hermite polynomial of degree 5, $H_5'(x) =$ ____.
   (a) $10H_4(x)$ (b) $5H_4(x)$ (c) $10H_6(x)$ (d) $5H_5(x)$
8. The Z-transform, $Z[(-1)^n](z) =$ ____.
   (a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{1}{z+1}$ (d) $\frac{1}{z-1}$

Q-2 Attempt *Any Seven* of the following: [14]

(a) State Dirichlet theorem for convergence of Fourier series.

(b) Write the formula for Fourier coefficients of the Fourier series of a $2L$-periodic function $f \in L^1[-L, L]$.

(c) For a 2π-periodic function f, state the relation between complex Fourier coefficients c_n of f and Fourier coefficients a_n, b_n of f.

(d) State and prove Parseval's identity for Fourier series.

(e) Let $u(x,t)$ be a function of two variables and let $u(x,t)$ and $u_x(x,t)$ both tend to 0 as $|x| \to \infty$. Show that $F_c[u_x](s) = -\sqrt{\frac{2}{\pi}}u(0,t) + sF_s[u](s)$.

(f) In usual notations, show that $L[t^n f(t)](s) = (-1)^n \frac{d^n}{ds^n} L[f](s)$.

(g) Define Heaviside function and compute its Laplace transform.

(h) Compute the Z-transform of $(\cos(\alpha n))_{n \geq 0}$, where $\alpha \in \mathbb{R}$.

(i) Find $H_2(x)$ and hence evaluate $H_2(0)$, notations being usual.

(P. T. O.)

Q-3 (a) Compute the half range Fourier sine series of $f(x) = \pi x - x^2$, $0 < x < \pi$. Use Parseval's identity to evaluate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$. [6]

(b) Compute the Fourier series of a 2π-periodic function $f(x) = x \sin x$, $-\pi \leq x \leq \pi$. Hence evaluate $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$. [6]

OR

(b) Applying Fourier series methods, solve $u_{xx} + u_{yy} = 0$, $0 < x < a$, $0 < y < b$ subject to $u(0, y) = u(a, y) = 0$ for all y, $u(x, b) = 0$ for all x, and $u(x, 0) = f(x)$ for all x. (You may assume both the functions equal to $-\lambda^2$ at the time of separation of variables). [6]

Q-4 (a) For $a > 0$, compute the Fourier transform of e^{-ax^2}. [6]

(b) Using Fourier integral representation of f, evaluate the integral $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right)\, dx$. [6]

OR

(b) Solve $u_{xx} + u_{yy} = 0$ $(x \in \mathbb{R},\ y > 0)$ subject to the conditions $u(x, 0) = f(x)$ $(x \in \mathbb{R})$, u is bounded as $y \to \infty$, both u and $\partial u/\partial x \to 0$ as $|x| \to \infty$ using Fourier transform methods. [6]

Q-5 (a) Using methods of Laplace transform, solve $u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$ subject to $u(0, t) = 0 = u(1, t)$ for all t, $u(x, 0) = \sin \pi x$ and $u_t(x, 0) = -\sin \pi x$, for all x. [6]

(b) Compute the inverse Laplace transform of the functions $\frac{1}{s(s^2+4)}$ and $\frac{1}{(s+a)(s+b)}$. [6]

OR

(b) Applying Laplace transform, solve $y'' + 9y = \cos 2t$ subject to $y(0) = 1 = y'(0)$. [6]

Q-6 (a) Find the Green's function for $y''(x) = f(x)$ subject to $y(0) = y(1) = 0$ and hence find the solution of the above equation when $f(x) = x^2$. [6]

(b) State Gram-Schmidt orthonormalization theorem. Orthonormalize the set $\{1, x, x^2\}$ over the interval $[-1, 1]$. [6]

OR

(b) i. Show that Z-transform is linear. [2]

ii. Using Z-transform methods, find the 100^{th} term of the Fibonacci sequence. [4]