S I C

SEAT No.________

[38 & A-9]

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester III
Wednesday, 01 November 2017
10.00 a.m. to 1.00 p.m.
PS03CMTH01 - Real Analysis II

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) Let μ_1 and μ_2 be measures on $(X, \mathscr{A})$. Which of the following need not be a measure?

(a) $e\mu_1 + \pi\mu_2$ (b) $e^{-1}\mu_1$ (c) $\mu_1 - \mu_2$ (d) $\mu_1 + \mu_2$

(2) The Lebesgue measure space $(\mathbb{R}, \mathscr{M}, m)$ fails to be ....

(a) σ- finite (b) saturated (c) complete (d) None of these

(3) Let δ_0 be the point mass measure at 0, and let m be the Lebesgue measure. Let $\nu = \delta_0 - m$. Then $\nu([-1,1] \cap (\mathbb{R} - \mathbb{Q})) =$ ....

(a) 1 (b) -1 (c) 2 (d) none of these

(4) Consider signed measure space $(\mathbb{N}, P(\mathbb{N}), \delta_1 - \eta)$, where η is the counting measure and δ_1 is Dirac measure concentrated at 1. Which of the following is a positive set of $\delta_1 - \eta$?

(a) $\{1\}$ (b) $\mathbb{N}$ (c) $\{1,2\}$ (d) $\mathbb{N} - \{1\}$

(5) Let η be the counting measure on $(\mathbb{N}, P(\mathbb{N}))$. Let $1 < p < r < \infty$. Which of the following is true?

(a) $L^p(\eta) \subset L^r(\eta)$ (b) $L^p(\eta) \supset L^r(\eta)$ (c) $L^p(\eta) = L^r(\eta)$ (d) none of these

(6) If f is a bounded measurable function, then which of the following true?

(a) f is essentially bounded (c) f is integrable
(b) f is square integrable (d) f is continuous

(7) Let η be the counting measure and δ_0 be the Dirac measure concentrated at 0. Which of the following is not an outer measure on $\mathbb{R}$?

(a) η (b) δ_0 (c) $\eta + \delta_0$ (d) none of these

(8) If F is the cumulative distribution of a Baire measure μ, then $\lim_{x \to -\infty} F(x)$ equals

(a) 0 (b) $-\infty$ (c) ∞ (d) $\mu(\mathbb{R})$

Q.2 Attempt any *Seven*. [14]

(a) Give an example of a function f such that f^2 is measurable but f is not measurable.
(b) Let $(X, \mathscr{A}, \mu)$ be a measure space, and let $E, F \in \mathscr{A}$ with $F \subset E$. Is it true that $\mu(E - F) = \mu(E) - \mu(F)$? Why?
(c) If f is integrable over X, then show that $|\int_X f d\mu| \leq \int_X |f| d\mu$.
(d) If ν is a signed measure on $(X, \mathscr{A})$ and $\alpha \geq 0$, then show that $(\alpha\nu)^+ = \alpha\nu^+$.
(e) If $\{A, B\}$ and $\{A_1, B_1\}$ are Hahn decompositions of ν, then show that $A \Delta A_1$ is a null set.

(f) If f is an essentially bounded function, then show that f is finite a.e. $[\mu]$ on X.
(g) State Riesz representation theorem.
(h) Let μ^* be an outer measure. If $E \subset X$ and $\mu^*(E) = 0$, then show that E is μ^*- measurable.
(i) If F is a cumulative distribution of a finite Baire measure μ, show that $\lim_{x\to\infty} F(x) = \mu(\mathbb{R})$.

Q.3

(a) Let $(X, \mathscr{A})$ be a measurable space. When is a map $f : X \to [-\infty, \infty]$ called measurable? Show that f is measurable if and only if $f^{-1}(\{-\infty\})$, $f^{-1}(\{\infty\})$ are measurable and that $f^{-1}(E)$ is measurable subset for every Borel subset E of $\mathbb{R}$. [6]

(b) Let $(X, \mathscr{A}, \mu)$ be a measure space, and let $\{f_n\}$ be a sequence of measurable functions converging f pointwise on X. If there is an integrable function g such that $|f_n| \leq g$ for all n, then show that the sequence $\{\int_X f_n d\mu\}$ converges to $\int_X f d\mu$. [6]

OR

(b) Let $(X, \mathscr{A}, \mu)$ be a measure space. If the functions f and g are integrable over X and if $\alpha, \beta \in \mathbb{R}$, then show that $\int_X (\alpha f + \beta g) d\mu = \alpha \int_X f d\mu + \beta \int_X g d\mu$. [6]

Q.4

(c) Suppose that ν is a signed measure on $(X, \mathscr{A})$ and $E \in \mathscr{A}$ satisfies $0 < \nu(E) < \infty$. Then show that there is a subset A of E such that A is a positive set and $\nu(A) > 0$. [6]

(d) Let ν and μ be σ- finite measures on a $(X, \mathscr{A})$, and let $\nu \ll \mu$. If f is a nonnegative measurable function on X, then show that $\int_E f d\nu = \int_E f[\frac{d\nu}{d\mu}] d\mu$ for every $E \in \mathscr{A}$. State the results you use. [6]

OR

(d) Let $(X, \mathscr{A}, \mu)$ be a measure space. Let $f : X \to [-\infty, \infty]$ be a measurable function such that either $\int_X f^+ d\mu < \infty$ or $\int_X f^- d\mu < \infty$. Define $\nu(E) = \int_E f d\mu$ for all $E \in \mathscr{A}$. Find Hahn decomposition and Jordan decomposition of ν. [6]

Q.5

(e) Define $L^\infty(\mu)$. Define essential supremum norm $\|\cdot\|_\infty$ on $L^\infty(\mu)$. Show that $(L^\infty(\mu), \|\cdot\|_\infty)$ is complete. [6]

(f) Let $(X, \mathscr{A}, \mu)$ be a finite measure space, let $1 \leq p < \infty$, and let q be such that $\frac{1}{p} + \frac{1}{q} = 1$. If g is an integrable function on $(X, \mathscr{A}, \mu)$ satisfying $\left|\int_X g\varphi d\mu\right| \leq M\|\varphi\|_p$ for some $M > 0$ and for all measurable simple functions φ, then show that $g \in L^q(\mu)$. [6]

OR

(f) Let $1 < p < \infty$. Show that $(L^p(\mu), \|\cdot\|_p)$ is a normed space. [6]

Q.6

(g) Define outer measure. When is a set called measurable with respect to outer measure. If μ^* is an outer measure on X and $\mathbb{B}$ is the σ- algebra of all μ^*- measurable subsets of X, then show that $(X, \mathbb{B}, \mu^*_{|\mathbb{B}})$ is a complete measure space. [6]

(h) Let μ be a measure on an algebra $\mathcal{A}$ of subsets of X. Define $\mu^* : P(X) \to [0, \infty]$ by $\mu^*(A) = \inf\{\sum_i \mu(E_i) : A \subset \bigcup_i E_i,\ \{E_i\} \subset \mathcal{A}\}$. Prove or disprove that μ^* is an outer measure on X. [6]

OR

(h) Let $\mathscr{A}$ be an algebra of subsets of X. Define $\mathscr{A}_\sigma$- set and $\mathscr{A}_{\sigma\delta}$- set. If μ is a σ- finite measure on an algebra $\mathcal{A}$ of subsets of X and μ^* is the outer measure induced by μ, then show that a subset E of X is (μ^*)- measurable if and only if E can be expressed as a difference $E = A - B$, where A is an $\mathcal{A}_{\sigma\delta}$- set and $\mu^*(B) = 0$. [6]

ЊЊЊЊЊЊЊЊЊ

SC

SEAT No.________

No. of printed pages: 2

[324 A-11]

SARDAR PATEL UNIVERSITY

M.Sc. Mathematics, Semester - III

Friday, 3rd November, 2017

PS03CMTH02, Mathematical Methods - I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: Assume standard notations wherever applicable. Figures to the right indicate full marks of the respective question.

Q-1 Choose the most appropriate option for each of the following questions: [8]

1. The Fourier series of 2π-periodic function $f(x) = -x^3$, $-\pi < x \leq \pi$ contains _____.
   (a) both cosine and sine terms (c) only sine terms
   (b) only cosine terms (d) only a constant term
2. Which of the following functions satisfy Dirichlet conditions in the interval $(0, 1)$?
   (a) $e^{\frac{1}{x}}$ (b) $\chi_{\mathbb{Q}}$ (c) $\frac{\sin x}{x}$ (d) $\sin\left(\frac{1}{x}\right)$
3. If f is an odd integrable function then the Fourier sine transform of f is _____.
   (a) $-iF[f]$ (b) $iF[f]$ (c) $F[f]$ (d) $-F_c[f]$
4. If $x^2 f(x) \in L^1(\mathbb{R})$, then $F[x^2 f(x)](s) =$_____.
   (a) $\frac{d^2}{ds^2}F[f](s)$ (b) $-\frac{d^2}{ds^2}F[f](s)$ (c) $-s^2\frac{d^2}{ds^2}F[f](s)$ (d) $-F[f''](s)$
5. Let $H(t-a)$ denote the Heaviside function at $a > 0$. Then $L[H(t-a)](s) =$_____.
   (a) $\frac{e^{-sa}}{s}$ (b) $-\frac{e^{-sa}}{s}$ (c) $\frac{e^{-sa}}{a}$ (d) $-\frac{e^{-sa}}{a}$
6. $L^{-1}\left[\frac{1}{(s-a)^2+b^2}\right](t) =$ _____.
   (a) $ae^{at}\sin bt$ (b) $be^{at}\sin bt$ (c) $\frac{1}{a}e^{bt}\sin at$ (d) $\frac{1}{b}e^{at}\sin bt$
7. Let $H_n(x)$ be Hermite polynomial of degree n. If $H_2(x) = 4x^2 - 2$, then $H_3{}'(0) =$_____.
   (a) -12 (b) 12 (c) -6 (d) 0
8. The domain of convergence of Z-transform of the sequence $(1, 0, 0, \ldots)$ is _____.
   (a) $|z| < 1$ (b) $|z| > 1$ (c) $\mathbb{C} \setminus \{0\}$ (d) $\mathbb{C}$

Q-2 Attempt ***Any Seven*** of the following: [14]

(a) State Dirichlet conditions on $[-\pi, \pi]$ for a 2π-periodic function $f : \mathbb{R} \to \mathbb{C}$.

(b) Show that the set $\{\cos nx \mid n \in \mathbb{N} \cup \{0\}\}$ is orthogonal over $[-\pi, \pi]$.

(c) Compute the half-range Fourier cosine series of a 2-periodic function $f(x) = 1$, $0 < x < 1$.

(d) If $f \in L^2(\mathbb{R}) \cap L^1(\mathbb{R})$, $F[f] \in L^2(\mathbb{R})$, then show that $\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F[f](s)|^2 ds$.

(e) If $f \in L^1(0, \infty)$ is twice differentiable and both $f(x), f^{(1)}(x) \to 0$ as $x \to \infty$, then show that $F_s[f^{(2)}](s) = s\sqrt{\frac{2}{\pi}}f(0) - s^2 F_s[f](s)$.

(f) In usual notations, show that $L\left[\frac{f(t)}{t}\right](s) = \int_s^{\infty} L[f](u)\, du$.

(g) Compute the Laplace transform of $\cos^2(2t)$.

(h) Compute the Z-transform of $(\sinh(\alpha n))_{n \geq 0}$, where $\alpha \in \mathbb{R}$.

(i) Compute the inverse Z-transform of $\dfrac{z^2}{(z-2)(z-3)}$.

Q-3 (a) Compute the Fourier series of a 4-periodic function $f(x) = x^2$, $-2 < x \le 2$. Use the series to compute the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$. [6]

(b) Compute the complex Fourier series of $f(x) = \begin{cases} x, & -\pi < x \le 0; \\ \pi - x, & 0 < x \le \pi. \end{cases}$ [6]

OR

(b) For some $L > 0$, solve $y_{tt} = c^2 y_{xx}$ subject to $y(0,t) = y(L,t) = 0$ for all $t > 0$, $y_t(x,0) = 0$ for $0 \le x \le L$, and $y(x,0) = mx(L-x)$ for all $x \in [0, L]$. (You may take both functions to be a negative constant while applying separation of variables). [6]

Q-4 (a) Solve $u_{tt} = c^2 u_{xx}$ ($x \in \mathbb{R}$, $t > 0$) subject to $u(x,0) = f(x)$ and $u_t(x,0) = 0$ for all $x \in \mathbb{R}$ and both $u, u_x \to 0$ as $|x| \to \infty$ using Fourier transform methods. [6]

(b) Compute the Fourier sine transform of $f(x) = e^{-\beta x}$, $x > 0$, $\beta > 0$. Also show that $\int_0^{\infty} \frac{x \sin(ux)}{\beta^2 + x^2}\, dx = \frac{\pi}{2} e^{-\beta u}$, $(u > 0)$. [6]

OR

(b) If $f, g \in L^1(\mathbb{R})$, in usual notations, show that $f * g \in L^1(\mathbb{R})$ and $F[f*g] = F[f]F[g]$. [6]

Q-5 (a) Using methods of Laplace tranform, solve $u_t = u_{xx}$, $0 < x < 1$, $t > 0$ subject to $u(0,t) = u(1,t) = 1$ for all t and $u(x,0) = 1 + \sin \pi x$, $0 < x < 1$. [6]

(b) Compute the inverse Laplace transform of the functions $\frac{1}{s(s+1)^4}$ and $\frac{1}{(s-1)(s^2+1)}$. [6]

OR

(b) If f is n-times differentiable and $f^{(r)}(t)e^{-st} \to 0$ as $t \to \infty$ for $r = 0, \ldots, n-1$, then show that $L[f^{(n)}](s) = s^n L[f](s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$. Hence solve $y^{(4)} - y = 1$ subject to $y(0) = y'(0) = y''(0) = y'''(0) = 0$. [6]

Q-6 (a) Find the Green's function for $y''(x) + y(x) = f(x)$ subject to $y(0) = y'(1) = 0$ and hence find its solution when $f(x) = 2x$. [6]

(b) In usual notations, show that $\int_{-\infty}^{\infty} H_n(x) H_m(x) e^{-x^2}\, dx = \begin{cases} 0, & m \ne n \\ 2^n n! \sqrt{\pi}, & m = n. \end{cases}$ [6]

OR

(b) i. Find a polynomial $p(x)$ of degree 2 so that $\int_{-1}^{1} |p(x) - \cos x|^2\, dx$ is minimum. [3]

ii. Using Z-transform methods, find the 15^{th} term of the Fibonacci sequence. [3]

########################

Sc

SEAT No.________

[39]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester III) Examination

Date: 16-11-2017 **Time: 10.00 To 01.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH01 – (Functional Analysis II)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) Which of the following sets is not a convex subset of a nls X?

(a) $\{x \in X, \|x\| < 1\}$ (c) $\{x \in X, \|x\| = 1\}$

(b) $\{x \in X, \|x\| \le 1\}$ (d) none of these

(2) For $F \in BL(X,Y)$, let $\alpha = \sup\{\|F(x)\| : x \in X, \|x\| = 1\}$ and $\beta = \sup\{\|F(x)\| : x \in X, \|x\| < 1\}$. Then

(a) $\alpha > \|F\|$ (b) $\beta < \|F\|$ (c) $\alpha > \beta$ (d) $\alpha = \beta$

(3) The space K^2 is strictly convex with the norm

(a) $\|\cdot\|_\infty$ (b) $\|\cdot\|_2$ (c) $\|\cdot\|_1$ (d) none of these

(4) Let Y be a closed subspace of a nls X. Then X is a Banach space if and only if

(a) Y & X/Y both are Banach spaces (c) Y is a Banach space

(b) X/Y is a Banach space (d) none of these

(5) Let X, Y be nlspaces and $F: X \to Y$ be linear. Then

(a) F is continuous $\Rightarrow$ F is closed (c) F is continuous $\Leftrightarrow$ F is closed

(b) F is closed $\Rightarrow$ F is continuous (d) none of these

(6) If I denote the identity operator on X, then $\sigma(I) =$

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) ϕ

(7) Let $F \in BL(X,Y)$ and F' be the transpose of F. Then

(a) $\|F\| > \|F'\|$ (b) $\|F\| < \|F'\|$ (c) $\|F\| = \|F'\|$ (d) none of these

(8) Let X be a nls. Then $x_n \xrightarrow{\|\cdot\|} x \Leftrightarrow x_n \xrightarrow{w} x$ in X, only if X =

(a) C[a, b] (b) l^p $(2 < p < \infty)$ (c) l^2 (d) l^1

2. Attempt any SEVEN: [14]

(a) Let Y be a subspace of a nls X. Prove that, if $Y^0 \ne \phi$, then Y = X.

(b) For $X = K^n$, show that $\|x\|_1 \le n\|x\|_\infty$, $\forall x \in X$.

(c) Let X and Y be nlspaces. Prove that, if $F: X \to Y$ is linear and continuous at 0, then F is bounded on $\bar{U}(0, r)$ for some $r > 0$.

(d) State Hahn-Banach extension theorem.

(e) Let X be a nls and $x \in X$. Define $j_x: X' \to K$ by $j_x(f) = f(x)$. Show that $\|j_x\| = \|x\|$.

(f) Define $F_n: (c_{00}, \|\cdot\|_\infty) \to K$ by $F_n(x) = \sum_{j=1}^{n} x(j)$. Then, find $\|F_n\|$.

(g) For $A \in BL(X)$, show that $\sigma_a(A) \subset \sigma(A)$.

(h) Let $F, G \in BL(X,Y)$. Prove that $(F + G)' = F' + G'$.

(i) Prove: If $\{x'_n\}$ is a sequence in X', then $x'_n \xrightarrow{w} x' \Rightarrow x'_n \xrightarrow{w^*} x'$ in X'.

(P.T.O.)

3. (a) Let Y be a closed subspace of a nls X. For $x + Y \in X/Y$, define $|||x + Y||| = \inf\{\|x + y\| : y \in Y\}$. Show that $|||\cdot|||$ defines a norm on the quotient space X/Y. [6]

(b) State and prove Holder's inequality. [6]

OR

(b) Let $X = C^1[0,1]$ and $Y = C[0,1]$, both with sup norms. Define $F: X \to Y$ by $F(x) = x'$, $\forall x \in X$. Show that F is linear but not continuous. [6]

4. (a) State and prove the Uniform boundedness principle. [6]

(b) Let Y be a subspace of a nls X. Let $a \in X$ and $a \notin \bar{Y}$. Prove that there is $f \in X'$ such that $f|_Y = 0$, $f(a) = \text{dist}(a, \bar{Y})$ and $\|f\| = 1$. [6]

OR

(b) Let $X \neq \{0\}$ and Y be nlspaces. If Y is a Banach space, then show that the nls BL(X,Y) is a Banach space with the operator norm. [6]

5. (a) Let X and Y be Banach spaces and $F: X \to Y$ be linear. If F is closed and surjective, then prove that F is continuous and open. [6]

(b) Let X be a nls and $P: X \to X$ be a projection. Prove that P is a closed map if and only if Z(P) and R(P) are closed in X. [6]

OR

(b) For $A \in BL(X)$, define $\sigma_a(A)$ and show that $k \in \sigma_a(A)$ if and only if there is a sequence $\{x_n\}$ in X such that $\|x_n\| = 1\ \forall n \in \mathbb{N}$ and $\|A(x_n) - kx_n\| \to 0$ as $n \to \infty$. [6]

6. (a) Let $1 \le p \le \infty$ and $\frac{1}{p}+\frac{1}{q} = 1$. For fixed $y \in l^q$, define $f_y: l^p \to K$ by $f_y(x) = \sum_{j=1}^{\infty} x(j)y(j)$. Then, show that $f_y \in (l^p)'$ and $\|f_y\| = \|y\|_q$. [6]

(b) Define weak convergence in a nls X. Show that if $\{x_n\}$ is a sequence in X, then $x_n \xrightarrow{\|\cdot\|} x \implies x_n \xrightarrow{w} x$ in X. [6]

OR

(b) Let X be a separable nls. Prove that every bounded sequence in X′ has a weak* convergent subsequence. [6]

X-X-X-X-X-X

SC

[107] SEAT No.________ No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - III Examination
Thursday, 09^{th} November, 2017
PS03EMTH02, Banach Algebras

Time: 10:00 a.m. to 01:00 p.m. Maximum marks: 70

Note: Figures to the right indicate full marks of the respective questions. Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) The algebra _____ does not have identity.
(i) $\ell^1(\mathbb{N})$ (ii) $L^1(\Gamma)$ (iii) $\ell^1(\mathbb{Z})$ (iv) $C_b(\mathbb{R})$

(b) The algebra _____ does not have divisor of zero.
(i) $C_c(\mathbb{R})$ (ii) $A(\mathbb{D})$ (iii) $C(\mathbb{D})$ (iv) $C_b(\mathbb{R})$

(c) Radical of $\mathbb{C}^2$ with the multiplication defined by $(x_1,x_2)(y_1,y_2) =$ _____, $((x_1,x_2),(y_1,y_2) \in \mathbb{C}^2)$, is $\mathbb{C}^2$.
(i) $(x_2y_2, 0)$ (ii) $(x_1y_1, 0)$ (iii) (x_1y_1, x_2y_2) (iv) (x_1y_2, x_2y_2)

(d) In the algebra _____, the spectrum of an element need not be compact.
(i) $C(\mathbb{R})$ (ii) ℓ^∞ (iii) $\ell^1(\mathbb{Z}_5)$ (iv) $A(\mathbb{D})$

(e) _____ is simple.
(i) $C_0(\mathbb{R})$ (ii) $C[0,1]$ (iii) $C^1[0,1]$ (iv) $M_2(\mathbb{C})$

(f) _____ is not a C^*-algebra.
(i) $M_n(\mathbb{C})$ (ii) $C_0(\mathbb{R})$ (iii) $C[0,1]$ (iv) $C^1[0,1]$

(g) If the Gel'fand map of a commutative unital Banach algebra $\mathcal{A}$ is isomorphism, then $\mathcal{A}$ is _____.
(i) a C^*-algebra (ii) *semisimple* (iii) $C[0,1]$ (iv) $C^1[0,1]$

(h) For a selfadjoint element x of a C^*-algebra, $r(x)$ _____.
(i) $< \|x\|$ (ii) $= \|x\|$ (iii) $> \|x\|$ (iv) > 0

Q-2 Attempt *Any Seven* of the following: [14]

(a) Prove the continuity of multiplication on a Banach algebra.

(b) Find all invertible elements of $\mathbb{C}^2$ with pointwise oerations.

(c) If a Banach algebra $\mathcal{A}$ has the identity e, then show that $\|e\| \geq 1$.

(d) Define ℓ^∞ and for $x, y \in \ell^\infty$, prove that $\|xy\|_\infty \leq \|x\|_\infty \|y\|_\infty$.

(e) Define *spectrum* of an element of an algebra. Find the spectrum of $2 + 3i \in \mathbb{C}$ as an algebra over $\mathbb{C}$.

(f) Define and give an example of a *complex homomorphism* on a Banach algebra.

(g) Define and give an example of a *maximal left ideal* of a Banach algebra.

(h) Define and give an example of a *Banach* algebra*.

(i) State Gel'fand-Naimark theorem for commutative C^*-algebras.

(C.P.T.O.)

Q-3 (j) Prove the completeness of $(C[0,1], \|\cdot\|_\infty)$. [6]

(k) State and prove the Gel'fand Mazur theorem. [6]

OR

(k) Show that the set of all invertible elements of a unital Banach algebra is open. [6]

Q-4 (l) State and prove the spectral radius formula in a Banach algebra. [6]

(m) Show that the spectrum of an element of a complex unital Banach algebra is nonempty. [6]

OR

(m) Define *radical* of a unital Banach algebra. Show that radical is a two sided ideal. [6]

Q-5 (n) Show that two complex homorphisms of a unital Banach algebra cannot have the same kernel. [6]

(o) Giving all details, show that the Gel'fand transform of a commutative unital Banach algebra is norm decreasing. [6]

OR

(o) Characterize all the closed ideals of $C[0,1]$. [6]

Q-6 (p) Show that spectrum of a selfadjoint element of a complex unital C^*-algebra is real. Show that the result does not hold in a Banach* algebra. [6]

(q) State and prove the Banach Stone theorem. [6]

OR

(q) For a normal element x of a C^*-algebra, prove that $\|x^2\| = \|x\|^2$. [6]

##########################

— x —

(2)

Sc

[91] SEAT No.________

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester III
Tuesday, 07 November 2017
10.00 a.m. to 1.00 p.m.
PS03EMTH03 - Problems and Exercises in Mathematics II

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) The value of $\lim_{n\to\infty}(2^n + 3^n + 4^n)^{\frac{1}{n}}$ is

(a) 1 (b) 2 (c) 3 (d) 4

(2) Let $f : [a, b] \to \mathbb{R}$ be a nonnegative continuous function. Which of the following is not true?

(a) If $f = 0$, then $\int_a^b f(x)dx = 0$
(b) If $\int_a^b f(x)dx = 0$, then $f = 0$.
(c) $f = 0$ if and only if $\int_a^b f(x)dx = 0$.
(d) none of these

(3) Let $f(z) = \frac{\cot z}{z^4}$. Then 0 is a pole of f of order

(a) 3 (b) 4 (c) 5 (d) 6

(4) 0 is .... of $\frac{1}{\sin\frac{1}{z}}$

(a) a non isolated singularity
(b) a removable singularity
(c) a pole
(d) an essential singularity

(5) Let τ, σ and u be the lower limit topology, upper limit topology and standard topologies on $\mathbb{R}$ respectively. Which of the following is a homeomorphic pair?

(a) $(\mathbb{R}, u), (\mathbb{R}, \sigma)$ (b) $(\mathbb{R}, u), (\mathbb{R}, \tau)$ (c) $(\mathbb{R}, \tau), (\mathbb{R}, \sigma)$ (d) none of these

(6) Consider the subspace $Y = \{(x, -x) : x \in \mathbb{R}\}$ of $\mathbb{R}_\ell \times \mathbb{R}_\ell$. Let $\mathfrak{F}$ be the collection of all continuous functions from $\mathbb{R}$ to Y. Then $\mathfrak{F}$ is .... set

(a) a finite (b) a countable (c) an uncountable (d) an empty

(7) What could be the maximum order of an element in A_{10} ?

(a) 10 (b) 21 (c) 5 (d) none of these

(8) Let G be a group of order n. Then G is abelian if

(a) $n = 21$ (b) $n = 36$ (c) $n = 15$ (d) none of these

Q.2 Attempt any *Seven*. [14]

(a) Let $0 < \delta < 1$, and let $f_n(x) = x^n$. Show that (f_n) converges uniformly on $[0, \delta]$.
(b) Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{k^2+n^2}$.
(c) Write the set of continuity of the map $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \cos x$ if $x \in \mathbb{Q}$ and $f(x) = \sin x$ if $x \in \mathbb{R} \setminus \mathbb{Q}$.
(d) Show that the range of a nonconstant entire function is dense in $\mathbb{C}$.
(e) Evaluate $\int_0^{2\pi}\exp(e^{i\theta} - \theta)d\theta$.
(f) Show that $\mathbb{R}_\ell$ is totally disconnected.
(g) Let τ and σ be topologies on X and $\tau \supset \sigma$. Suppose that (X, τ) is compact. Is (X, σ) compact? Justify.

(P.T.O.)

(h) What is the smallest composite integer $n > 1$ such that there is a unique group of order n?

(i) Let $\sigma \in S_7$ with $\sigma^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$. Find σ.

Q.3

(a) Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function satisfying $f(x^2) = f(x)$ for all $x \in [0, \infty)$. Show that f is a constant map. Is the same true if f is not continuous? Why? [6]

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be a bounded function. For x_0 and $\delta > 0$ define $m_\delta(x_0) = \inf_{x \in (x_0-\delta, x_0+\delta)} f(x)$ and $M_\delta(x_0) = \sup_{x \in (x_0-\delta, x_0+\delta)} f(x)$. Let $m(x_0) = \sup_{\delta>0} m_\delta(x_0)$ and $M(x_0) = \inf_{\delta>0} M_\delta(x_0)$. Show that f is continuous at x_0 if and only if $m(x_0) = M(x_0)$. [6]

OR

(b) Suppose that $f : [a, b] \to \mathbb{R}$ is continuous and f is differentiable on (a, b) except possibly a point $c \in (a, b)$. If $\lim_{x\to c} f'(x) = \ell$, then show that f is differentiable at c. Also, show that if $g : \mathbb{R} \to \mathbb{R}$ is differentiable and g' is bounded, then g is uniformly continuous. [6]

Q.4

(c) Let D be a domain and let $f, g : D \to \mathbb{C}$ be analytic. Prove the following statements. [6]

(क) If $fg = 0$, then either $f = 0$ or $g = 0$.

(ख) If $f^n = g^n$ for some $n \in \mathbb{N}$, then $f = \alpha g$ for some $\alpha \in \mathbb{C}$ with $|\alpha| = 1$.

(d) Let f and g be analytic at z_0. If z_0 is a zero of f order k and z_0 is a zero of g of order $k+2$, then show that z_0 is a double pole of $\frac{f}{g}$. Also, find the residue of $\frac{f}{g}$ at z_0. State the results you use. [6]

OR

(e) Let f be an entire function such that $f(z+1) = f(z)$ and $f(z+i) = f(z)$ for all $z \in \mathbb{C}$. Prove that f is a constant map. [6]

Q.5

(f) Let X be any set. Let τ and σ be topologies on X such that X is compact and Hausdorff with respect to both the topologies. If τ and σ are comparable, then show that they are equal. Also, prove that any bijective continuous map from a compact space to a Hausdorff space is a homeomorphism. [6]

(f) Let $K = \{\frac{1}{n} : n \in \mathbb{N}\}$. Define K- topology on $\mathbb{R}$. Show that $\mathbb{R}$ with K- topology is Hausdorff but not regular. [6]

OR

(g) Let $\{X_\alpha\}$ be a collection topological space. Define the product topology and the box topology on the product space. Let $\mathbb{R}^\omega$ be the countable infinite product of $\mathbb{R}$ with itself. Let $f : \mathbb{R} \to \mathbb{R}^\omega$ be $f(t) = (t, 2t, 3t, \ldots)$. Show that f is continuous if $\mathbb{R}^\omega$ has the product topology and f is discontinuous if $\mathbb{R}^\omega$ has the box topology. [6]

Q.6

(h) Find all subgroups of order 5 in $\mathbb{Z}_{10} \oplus \mathbb{Z}_{10}$. [6]

(i) Let $A = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}$. Find A^{100}. [6]

OR

(h) Let $P_2 = \{p(x) \in \mathbb{R}[x] : \deg(p(x)) \leq 2\}$. Define $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$ where $p, q \in P_2$. Then find an orthonormal basis of P_2. [6]

卐卐卐卐卐卐卐卐

— × —

S.C

[92/A-32] Seat No. ________ No. of printed pages: 02

SARDAR PATEL UNIVERSITY
M.Sc. (Mathematics) (III Semester) Examination
Tuesday, 7 November 2017
10.00 am - 1.00 pm
PS03EMTH16 - Relativity - I

Total Marks : 70

Note: 1. Answer to all questions to be given in the answer book only.
2. Figures on the right indicate full marks.

Q-1 Choose appropriate answer from the options given. (08)

1. In General Galelian transformation time coordinate is ________.
   (a) relative (b) absolute (c) not defined (d) none of these
2. Maxwell's equations are not invariant under __________ transformation.
   (a) General Lorentz (b) General Lorentz
   (c) Special Galelian (d) None of these
3. A frame in rotational motion relative to an inertial frame is ______
   (a) an inertial frame (b) a special frame
   (c) a non-inertial frame (d) not a frame
4. Shape of an object is________ under Special Lorentz transformations
   (a) changes (b) does not change (c) non-deterministic (d) rectangular
5. In Special Relativity, moving clocks appear to run ________
   (a) slow (b) fast (c) with constant speed (d) uniformly
6. Velocity 4-vector is ___________.
   (a) space-like (b) of constant magnitude (c) null (d) contravariant
7. Which one of the following is not correct according to Special Relativity?
   (a) Mass is equivalent to energy.
   (b) Mass changes with motion.
   (c) Mass of a particle remains constant during the motion.
   (d) Rest mass and moving mass of a photon are different.
8. The type of Riemann curvature tensor is of ________ type.
   (a) (4,0) (b) (0,4) (c) (0,2) (d) (2,0)

Q-2 Attempt any SEVEN (14)

1. Show that Newton's equations are invariant under special Galelian transformation.
2. Why Michelson-Morley experiment was performed?
3. State the formula for relativistic composition of velocities.
4. Explain the meaning of length contraction.
5. When two events are said to be simultaneous?
6. Define a null vector.
7. State the expression of transformation of a contravariant vector.
8. What are various spacetime structures?
9. Show that gradient of a scalar is a covariant vector.

(P.T.O.)

Q-3

(a) State Maxwell's equations. Show that in vacuum they reduce to wave equations. (06)

(b) Derive the General Lorentz transformation. (06)

OR

(b) A rectangular sheet of length 100 cm and breadth 50 cm is moving along its length with velocity 0.6c. Find the apparent area of the sheet.

Q-4

(a) What is meant by aberration of light? Discuss it using special relativity. (06)

(b) Define spacetime interval and show that it is invariant under SLT. (06)

OR

(b) Sodium light is of wave length 5896 A°. Find the wave length shift in relativistic longitudinal Doppler effect if the observer approaches with the velocity 0.8c to the source of a sodium light.

Q-5

(a) Define velocity 4-vector. Show that it has constant norm. (06)

(b) In usual notations derive $T = (m - m_0)c^2$ and hence deduce $E = mc^2$. (06)

OR

(b) Rest mass of a body is 100 kg, its apparent mass is 125 kg. What is the velocity of the object? Also calculate its kinetic energy.

Q-6

(a) Derive geodesic equation. (06)

(b) State the transformation law for a tensor of type (2,0). Show that transformation satisfies group property. (06)

OR

(b) Discuss principle of equivalence and principle of covariance.

****

****

*****

SC

[93]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - III Examination

Saturday, 11^{th} November, 2017

PS03EMTH08, Group Theory

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: Figures to the right indicate full marks of the respective questions. Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) $A(S)$ is commutative if S has ____ elements.
(i) 2 (ii) 3 (iii) 4 (iv) 5

(b) There are ____ subgroups of S_3 which are not normal in S_3.
(i) 2 (ii) 3 (iii) 4 (iv) 5

(c) In a group of order 72, c_e = ____.
(i) 1 (ii) 8 (iii) 9 (iv) 72

(d) For subgroups H, K of a group G, if $o(H)$ = ____ and $o(K)$ = ____, then HK is cyclic.
(i) 2, 2 (ii) 2, 3 (iii) 3, 3 (iv) 3, 5

(e) The smallest possible order of a group G in which converse of the Lagrange's theorem fails is ____.
(i) 3 (ii) 6 (iii) 9 (iv) 12

(f) The order of 5-Sylow subgroups of S_{25} is ____.
(i) 5^6 (ii) 6^5 (iii) 5^2 (iv) 2^5

(g) If $G = N_1 \cdot N_2 \cdot N_3$, a direct product, then N_2 ____.
(i) is nonabelian (ii) $\cap(N_1 \cdot N_3) = \{e\}$ (iii) is abelian (iv) is finite

(h) Number of nonisomorphic abelian groups of order 672 is ____.
(i) 5 (ii) 7 (iii) 9 (iv) 32

Q-2 Attempt ***Any Seven*** of the following: [14]

(a) On $\mathbb{R}$, prove the associativity of the operation $x \odot y = (x^3 + y^3 - 1)^{1/3}$, $(x, y \in \mathbb{R})$.

(b) Let H be a normal subgroup of an infinite group G. For $a, b \in G$, define the relations $a \equiv b \mod H$ if $ab^{-1} \in H$ and $a \sim b \mod H$ if $a^{-1}b \in H$. Show that $a \equiv b \mod H \Rightarrow a \sim b \mod H$.

(c) Find all conjugates of $(1, 2)(2, 4)$ in S_4

(d) For $G = (\mathbb{R}, +)$, show that $\text{Aut}(G)$ is a subgroup of $A(G)$.

(e) If there are more than two p-Sylow subgroups of a finite group G, then show that none of them can be normal.

(f) Let G be a group and $\mathcal{P}(G)$ be the power set of G. For $M, N \in \mathcal{P}(G)$, define $M \sim N$ if there is $g \in G$ such that $M = Ng$. Show that $\sim$ is an equivalence relation on $\mathcal{P}(G)$.

(g) For a group G and $s \in \mathbb{N}$, let $G(s) = \{x \in G : x^s = e\}$. Show that $G(s)$ need not be a subgroup of G.

(1) (P.T.O.)

(h) Define the term *invariants* and list all possible invariants of groups of order 81.

(i) Let $A = \{\theta \in S_{20} : i\theta = i \text{ for all } i \geq 17\}$. Show that $A \approx S_{16}$.

Q-3 (j) State and prove Cayley's theorem. [6]

(k) Let H, K be two subgroups of a finite group G. Prove that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$. [6]

OR

(k) Let H, K be subgroups of a group G. Show that HK is a subgroup if and only if $HK = KH$. [6]

Q-4 (l) For a prime $p \in \mathbb{N}$, let G be a group of order p^4. Show that $o(Z(G)) \neq p^3$. [6]

(m) Show that the relation defined on a group G as $x \sim y$ if $x = gyg^{-1}$ for some $g \in G$, is an equivalence relation. Using this or otherwise, prove that for $a \in G$, the sets $C(a)$ and $G/N(a)$ are in one to one correspondence. [6]

OR

(m) Define *inner automorphism of a group* G and prove that $\mathscr{I}(G) \approx G/Z(G)$. [6]

Q-5 (n) Define the term *double coset*. Let G be a subgroup of a finite group M and $p \in \mathbb{N}$ be a prime such that $p \mid o(G)$. If M has a p-Sylow subgroup P, then show that G has a p-Sylow subgroup Q of the form $P = G \cap (xQx^{-1})$ for some $x \in M$. [6]

(o) (i) Prove that a group of order 1225 is abelian. [6]

(ii) Prove that a group of order 200 cannot be simple.

OR

(o) Let G be a group of order 30. If 5-Sylow subgroup of G is not unique, then find the number of elements of order 5 in G. [6]

Q-6 (p) Let $\{H_i : 1 \leq i \leq n\}$ be a family of groups and $G = H_1 \times H_2 \times \cdots \times H_n$ be the group with pointwise operations. For $1 \leq k \leq n$, define $\overline{H_k} = \{x = (x_i) \in G : x_i = e_i, \text{ if } i \neq k\}$. Show that for all $h \in G$, there are unique $h_i \in \overline{H_i}$ such that $g = h_1 \cdot h_2 \cdot \cdots \cdot h_n$. Is $\overline{H_2}$ normal in G? Justify. [6]

(q) If p is a prime integer and $n \in \mathbb{N}$, then show that every abelian group of order p^n can be written as a direct product of cyclic subgroups. [6]

OR

(q) If abelian groups G and G' have the same invariants, then show that $G \approx G'$. [6]

(2)

(41) SEAT No.________ SC

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-III), PS03EMTH11, Mathematical Probability Theory;
Tuesday, 14^{th} November, 2017; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [08]

1. Let $(\Omega, \mathcal{A}, P)$ be probability space. Which one from following is not true ?
   (A) $P(A \setminus B) = P(A) - P(B),\ A, B \in \mathcal{A}$ (B) $P(\varphi) = 0$
   (C) $P(A) \leq P(B)$ if $B \supset A,\ A, B \in \mathcal{A}$ (D) none of these
2. The probabilities of A, B and C solving a problem are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. Then the probability that the problem will be solved is
   (A) $\frac{3}{4}$ (B) $\frac{29}{32}$ (C) $\frac{1}{3}$ (D) $\frac{3}{32}$
3. $P(X > x, Y > y) =$
   (A) $F_X(x) + F_Y(y) - F(x,y)$ (B) $1 - F_X(x) - F_Y(y) - F(x,y)$
   (C) $1 - F_X(x) - F_Y(y) + F(x,y)$ (D) none of these
4. Which one from the following is not true for a random variable?
   (A) The first moment is the expectation
   (B) The first central moment is zero
   (C) The second central moment is variance
   (D) none of these
5. $X_n \xrightarrow{P} 0 \Leftrightarrow$
   (A) $\lim_{n\to\infty} E(\frac{X_n}{1-X_n}) = 0$ (B) $\lim_{n\to\infty} E(X_n) = 0$
   (C) $\lim_{n\to\infty} E(X_n^2) = 0$ (D) none of these
6. $X_n \xrightarrow{P} c \Leftrightarrow \lim_{n\to\infty} F_n(x) = F(x)$ where
   (A) $F(x) = \begin{cases} 1 & ,x < c \\ 0 & ,x \geq c. \end{cases}$ (B) $F(x) = c$
   (C) $F(x) = \begin{cases} 0 & ,x < c \\ 1 & ,x \geq c. \end{cases}$ (D) none of these
7. If $\varphi(u)$ is characteristic function of random variable X, then the characteristic function of $1 - X$ is
   (A) $e^{-iu}\varphi(u)$ (B) $e^{iu}\varphi(-u)$ (C) $\varphi(-u)$ (D) $\varphi(1-u)$
8. Let F be a distribution function and h be corresponding characteristic function. For any $u > 0$, $\exists K > 0 \ni \frac{u}{K}\int_{|x|\geq\frac{1}{u}} dF(x)$
   (A) $\leq \int_0^u [h(0) - Re(h(v))]dv$ (B) $\geq \int_0^u [h(0) - Re(h(v))]dv$
   (C) $\leq \int_0^u [Re(h(v)) - h(0)]dv$ (D) none of these

(P.T.O.)

Q.2 Attempt any ***seven***: [14]

(a) Show that X is random variable on $(\Omega, \mathcal{A})$ iff $X^{-1}(\mathcal{E}) \subset \mathcal{A}$ where $\mathcal{E}$ is any collection of subsets of $\mathbb{R}$ which generates Borel σ-algebra.

(b) Define joint distribution function of random variables X and Y.

(c) State Jordan decomposition theorem.

(d) Let X_n be a sequence of random variables with $E(X_n) = 2$ and $\text{Var}(X_n) = \frac{1}{\sqrt{n}}$, $\forall n$. Does X_n converge in probability ? Justify.

(e) Show that $E(X+Y) = E(X) + E(Y)$ where X and Y are simple random variables.

(f) State Holder's and Minkowski's inequalities for random variables.

(g) State Weak Law of Large Numbers.

(h) State Levy's theorem.

(i) Show that characteristic function is continuous.

Q.3

(a) Let X be non-negative extended random variable on $(\Omega, \mathcal{A})$. Then show that $\exists$ an increasing sequence $\{X_n\}$ of non- negative simple random variables on $(\Omega, \mathcal{A})$ such that $X_n(\omega) \to X(\omega), \forall\, \omega \in \Omega$. [06]

(b) Find mean and variance of standard normal random variable. [06]

OR

(b) An experiment consists of three independent tosses of a fair coin. Let X denote the number of heads, Y denote the number of head runs and Z denote the length of head runs, a head run being defined as consecutive occurrence of at least two heads, its length being the number of heads occurring together in three tosses of the coin. Find pmf of (i) X, (ii) Y, (iii) Z, (iv) $X+Y$ (v) XY.

Q.4

(a) Show that $X_n \xrightarrow{L} X$ if $X_n \xrightarrow{P} X$. [06]

(b) If $X_n \xrightarrow{P} X$ and X_n is monotonic sequence then show that $X_n \xrightarrow{a.s.} X$ [06]

OR

(b) Prove or disprove: $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{L} Y$ then $X_n + Y_n \xrightarrow{L} X + Y$.

Q.5

(a) Let $\varphi(u)$ be the characteristic function of r.v. X and F be the distribution function of X. If $a, b (a < b)$ are points of continuity of F then show that [06]

$$F(b) - F(a) = \lim_{U\to\infty} \frac{1}{2\pi} \int_{-U}^{U} \frac{e^{-iua} - e^{-iub}}{iu} \varphi(u)du.$$

(b) Let $\{F_n\}$ be a sequence of distribution functions. Show that there exists a distribution function F and a subsequence $\{F_{n_k}\}$ of $\{F_n\}$ such that $F_{n_k} \to F$ weakly. [06]

OR

(b) Find probability density functions of random variables whose characteristic functions are (i) $e^{-\frac{1}{2}u^2}$ (ii) $e^{-|u|}$.

Q.6

(a) State and prove Central Limit Theorem. [06]

(b) State and prove Kolmogorav's inequality. [06]

OR

(b) State and prove Strong Law of Large Numbers.

*****

(108) SEAT No.________

No of printed pages: 4

SC

Sardar Patel University

M.Sc. (Sem-III), PS03EMTH12, Financial Mathematics-I;
Thursday, 9^{th} November, 2017; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) The required table of normal distribution is attached with this question paper; (ii) Calculator is allowed.

Q.1 Answer the following. [8]

1. How many types of basic financial derivatives are there?
   (A) 4 (B) 3 (C) 5 (D) 2
2. Futures contracts traded on
   (A) Over the counter market (B) Exchange traded market
   (C) On the counter market (D) none of these
3. How many types of repo rates are available?
   (A) 3 (B) 4 (C) 2 (D) 1
4. How much dollars would I pay now to receive \$110 after one year with the interest rate 10% per annum with semi annual compounding?
   (A) 100 (B) 105 (C) 120 (D) none of these
5. If X is the random variable giving the number of heads obtained in an experiment of tossing a coin three times, then $P(X < \pi)$ is
   (A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{3}{8}$ (D) none of these
6. The mean change per unit time is known as
   (A) repo rate (B) variance rate (C) drift rate
   (D) none of these
7. The solution of $y' + y = 0$ is
   (A) $e^{-x} + 1$ (B) e^{-x} (C) $\ln x$ (D) none of these
8. The delta for European call option is
   (A) $1 - N(-d_1)$ (B) $N(d_1) - 1$ (C) $N(d_2)$ (D) $N(-d_2) - 1$

Q.2 Attempt any *seven*: [14]

(a) Explain initial margin in the futures contract.
(b) Define over the counter market.
(c) Explain the difference between buying a call option and selling a put option.
(d) Define interest rate with an example.
(e) Define variance of random variable.
(f) State Itô's lemma.
(g) Give formula of two step binomial model.
(h) Define Unit Impulse function.
(i) What are the assumptions in the derivation of BSM differential equation?

(P.T.O.)

Q.3

(a) Discuss arbitrageur with an example. [6]

(b) Suppose that a put option with striking price £600 with option value £20 expires in next three months. Under what circumstances will the sellar of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option. [6]

OR

(b) A company enters into a short position in futures contract to sell 5000 bushels of wheat for 450 cents per bushel. The initial margin is $3000 and the maintenance margin is $2000. What price change would lead to a margin call ? Under what circumstances could $1500 be withdrawn from the margin account ?

Q.4

(a) Explain (i) repo rate (ii) n-year zero rate [6]

(b) Discuss markov process with an example. [6]

OR

(b) Suppose that spot interest rates with continuous compounding are as follows:

year	spot rate(%)
1	2.0
2	3.0
3	3.7
4	4.2
5	4.5

Calculate forward interest rates for $2^{nd}, 3^{rd}, 4^{th}$ and 5^{th} years.

Q.5

(a) Explain generalized one step binomial model. [6]

(b) Consider a variable S that follows process $dS = \mu S dt + \sigma S dz$ where z follows Wiener process. What is the process followed by (i) $\ln S$ (ii) S^2 (iii) e^S [6]

OR

(b) Find the price of a 6 months European put option with a strike price $42 on a stock whose current price is $40. Also the stock price either moves up or down by 10% for each 3 months period and the risk free interest rate is 12% per annum with continuously compounding.

Q.6

(a) Derive Put-Call Parity for European options. [6]

(b) Find delta for European put option. [6]

OR

(b) Find the value of an European call option on a non-dividend paying stock when the current stock price is £30, the strike price is £29, the risk free interest rate is 5% per annum with continuously compounding, the volatility is 25% per annum, and the time to maturity is 4-months.

*****

Table for $N(x)$ When $x \leqslant 0$

This table shows values of $N(x)$ for $x \leqslant 0$. The table should be used with interpolation. For example,

$$\begin{aligned} N(-0.1234) &= N(-0.12) - 0.34[N(-0.12) - N(-0.13)] \\ &= 0.4522 - 0.34 \times (0.4522 - 0.4483) \\ &= 0.4509 \end{aligned}$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(P.T.O.)

Table for $N(x)$ When $x \geqslant 0$

This table shows values of $N(x)$ for $x \geqslant 0$. The table should be used with interpolation. For example.

$$\begin{aligned} N(0.6278) &= N(0.62) + 0.78[N(0.63) - N(0.62)] \\ &= 0.7324 + 0.78 \times (0.7357 - 0.7324) \\ &= 0.7350 \end{aligned}$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Sc

[42] SEAT No.________ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-III) Examination

November – 2017

Tuesday, 14 November, 2017

Time: 10:00 AM to 01:00 PM

Mathematics **Course No.PS03EMTH13 (Operations Research)**

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) An LPP must have
(a) a linear objective function (b) at least one linear constraints
(c) at least three decision variables (d) all of these

(2) A constraint
(a) is always in an equation form (b) necessarily contains all variables
(c) imposes uncertainty of solution (d) none of these

(3) In the graphical method, optimal solution set is __________.
(a) always unbounded (b) convex (c) parallelogram (d) concave

(4) A basic feasible solution to a system is called _______________ if all of basic variables are non-zero.
(a) non-degenerate (b) non-basic
(c) degenerate (d) non-appropriate

(5) Which of the following is incorrect?
(a) transportation problem is an LPP (b) assignment problem is an LPP
(c) transportation problem is an NLPP (d) none of these

(6) If the availability is less than the demand in a TP then__________
(a) a dummy demand point zero cost is added.
(b) a dummy demand point with negative cost is added.
(c) a dummy source with zero cost is added.
(d) a dummy source with negative cost is added.

(7) Which of the following methods is useful to sole an NLPP?
(a) Method of Lagrange's multipliers (b) Big M method
(c) VAM (d) all of these

(8) In a non-linear programming problem
(a) there is only one decision variable
(b) all variables are unbounded
(c) at least one constraint is non-linear
(d) at least one constraint is of equality type

Q-2 Answer any **Seven.** (14)

(1) What is the meaning of a feasible solution?
(2) Give an example of a convex set.
(3) What is use of a surplus variable?
(4) What is meant by a degenerate solution?
(5) Define net evaluation in simplex method.
(6) How a transportation problem is balanced?
(7) Name any two methods for finding an IBFS to a transportation problem.
(8) Describe the meaning of an assignment problem.
(9) Explain drawbacks of using graphical method for solving an NLPP.

(P.T.O.)

Q-3

(a) Describe the steps in simplex method for solving an LPP. (06)

(b) Solve the following LPP using graphical method: (06)

Max $8000x_1 + 7000x_2$

Subject to

$3x_1 + x_2 \leq 66, x_1 + x_2 \leq 45, 0 \leq x_1 \leq 20, 0 \leq x_2 \leq 40$

OR

(b) Solve the following L.P.P. by Simplex Method :

Max $Z = 4000x_1 + 2000x_2 + 5000x_3$

subject to $12x_1 + 7x_2 + 9x_3 \leq 1260$, $22x_1 + 18x_2 + 16x_3 \leq 19008$,

$2x_1 + 4x_2 + 3x_3 \leq 396$ and $x_1, x_2, x_3 \geq 0$.

Q-4

(a) Using appropriate example explain handling of unrestricted variable in an LPP. (06)

(b) Obtain the dual of the following problem : (06)

Min $Z = 3x_1 - 2x_2 + 4x_3$

subject to $3x_1 + 5x_2 + 4x_3 \geq 7$, $6x_1 + x_2 + 3x_3 \geq 4, 7x_1 - 2x_2 + 4x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3, 4x_1 + 7x_2 - 2x_3 \geq 2$;$x_1, x_2, x_3 \geq 0$

OR

(b) Solve the following L.P.P. by using two-phase method :

Max $Z = 5x_1 + 3x_2$

subject to $2x_1 + x_2 \leq 1, x_1 + 4x_2 \geq 6$, and $x_1, x_2 \geq 0$.

Q-5

(a) Describe Hungarian method for an assignment problem. (06)

(b) Obtain an initial basic feasible solution to the following transportation problem using VAM. Also find the cost involved in the solution you obtained. (06)

	D1	D2	D3	D4	Availability
O1	5	1	3	3	34
O2	3	3	5	4	15
O3	6	4	4	3	12
O4	4	-1	4	2	19
Demand	21	25	17	17	

OR

(b) Solve the following assignment problem.

	A	B	C	D
I	3	4	11	9
II	5	7	8	9
III	5	6	6	7
IV	4	6	8	8

Q-6

(a) Derive Kuhn-Tucker condition for a general NLPP with m (<n) constraints. (06)

(b) Discuss graphical representation of the following NLPP

Max. $z = x_1^2 + x_2^2$ s. t. $x_2^2 - x_1 \leq 1$, $x_1 + x_2 \leq 2$; $x_1, x_2 \geq 0$

What can be said about the solution?

OR

(b) Obtain bordered Hessian matrix for the following NLPP:

Optimize $z = 4x_1 + 9x_2 - x_1^2 - x_2^2$ subject to the constraints

$4x_1 + 3x_2 = 15$, $3x_1 + 5x_2 = 14$ and $x_1, x_2 \geq 0$.

******

—x—

SC

SEAT No.______

[40]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-III), PS03EMTH21, Mathematics Education-I;
Thursday, 16^{th} November, 2017; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. Which one from the following books written by Fibonacci?
   (A) Arithmetica (B) Liber Abacci (C) Liber Fibonacci (D) none of these
2. Which of the following number system has base 60 ?
   (A) Hindu-Arabic (B) Egyptian (C) Babylonians (D) none of these
3. The value of Golden ratio is the solution of equation
   (A) $x^2 = x - 1$ (B) $x^2 = x + 1$ (C) $x^2 + x = 2$ (D) none of these
4. For $n \in \mathbb{N}$, $2^{2^n} + 1$ is a prime number. This conjectured was given by
   (A) Euler (B) Goldbach (C) Fermat (D) none of these
5. Which one from the following is a group ?
   (A) $(\mathbb{N}, +)$ (B) $(2\mathbb{N}, -)$ (C) $(\mathbb{N} \cup \{0\}, +)$ (D) none of these
6. 'Siddhanta Shiromani' written by
   (A) Aryabhatta (B) Brahmagupta (C) Bhaskaracharya (D) Ramanujan
7. The major contribution of Janos Bolyai in the field of
   (A) Modern algebra (B) Co-ordinate geometry (C) Euclidean geometry (D) Non-Euclidean geometry
8. $\sin(60^0 + \theta) - \cos(30^0 - \theta) =$
   (A) $2\cos\theta$ (B) 1 (C) 0 (D) $2\sin\theta$

Q.2 Attempt any *seven*: [14]

(a) Find the remainder when $(125)^{57}$ is divided by 7.
(b) State the fundamental theorem of arithmetic.
(c) Is 287 a prime number? justify.
(d) What is perfect number? Give an example of it.
(e) Find $\sqrt{1 + \sqrt{2 + \sqrt{2 + ...}}}$
(f) State Fermat's last theorem.
(g) State Euler's problem of 36 soldiers.
(h) Find the equation of a line which passes through (2,3) and makes an angle of 45^0 with the positive direction of x-axis.
(i) What is Euler formula for planer graph.

Q.3

(a) Discuss different periods of development of mathematics. [06]

(b) Prove or disprove: There are finitely many prime numbers. [06]

OR

(b) Write 1923 with (i) base 2 (ii) base 3 (iii) base 5 (iv) base 7.

Q.4

(a) Write any one biography from the following: [06]
(i) Fibonacci (ii) Ramanujan

(b) Discuss (i) Goldbach's conjecture (ii) Waring's conjecture (iii) Pigeonhole principle [06]

OR

(b) Let (a, b, c) be Pythagorian triplet with $a^2+b^2=c^2$ and P be a perimeter of Pythagorian triangle. Show that $P|ab$.

Q.5

(a) Write any one biography from the following: [06]
(i) Abel (ii) Galois

(b) Discuss contributions of Lagrange and Cauchy in the development modern algebra. [06]

OR

(b) Solve $x^3 - 6x = 9$ using cardan's method.

Q.6

(a) Discuss relation of arithmetic and algebra with solid geometry. [06]

(b) Discuss the crazy cube problem. [06]

OR

(b) If regular hexagon and circle having the same areas, then find the relation between its perimeters.

—X—

SC

(94)

SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester III) Examination

Date: 11-11-2017, Saturday **Time: 10.00 To 01.00 p.m.**
Subject: MATHEMATICS **Paper No. PS03EMTH23 – (Graph Theory – II)**
Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The number of spanning trees in K_n ($n > 1$) is
(a) n (b) n^2 (c) n^{n-2} (d) n^n

(2) If all the digits in the Pruffer code are same, then the graph is
(a) Star graph (b) Path graph (c) Cycle graph (d) $K_{n,n}$ ($n > 1$)

(3) The graph K_{11} can be decomposed into copies of
(a) $K_{1,6}$ (b) P_6 (c) P_5 (d) none of these

(4) If f is a flow on a network N = (V, A) with source s and sink t, then $f(\{s\},V) =$
(a) $f(V,\{s\})$ (b) $f(V,\{t\})$ (c) $f(\{t\}, V)$ (d) none of these

(5) Let A be a matrix with spectrum $\{-2, -1, 3, -3, 1\}$. Then det(A) =
(a) –2 (b) 9 (c) 18 (d) –18

(6) Let $G = K_{4,3}$. Then the non-zero eigen values for G is
(a) 2 (b) 3 (c) 4 (d) 12

(7) The Ramsey number R(p, q)
(a) $= \text{Min}\{p, q\}$ (b) $\leq \text{Min}\{p, q\}$ (c) $\geq \text{Max}\{p, q\}$ (d) $= \text{Max}\{p, q\}$

(8) If $E = \{1, 2, 3\}$ with $\mathcal{M} = \{\{3\}, \{2\}, \{2,3\}\}$ as hereditary system, then $r(\{1,2\}) =$
(a) 0 (b) 1 (c) 2 (d) 3

2. Attempt any SEVEN: [14]

(a) Construct a tree with Pruffer code (1234).

(b) Give one graceful labeling of P_6 with detail.

(c) Define a cut in a network and give one example of it.

(d) State Pigeonhole Principle.

(e) Prove: If G is k regular graph, then k is an eigen value of G.

(f) Prove: $sp(A^2) = \{\lambda^2 : \lambda \in sp(A)\}$.

(g) Define u-v separating set and give one example of it.

(h) Prove or disprove: R(3, 4) = 8.

(i) If $E = \mathbb{Z}$ with $\mathcal{M} = \{X \subset E; |X| < 7\}$ as hereditary system, then find $B_{\mathcal{M}}$ and $C_{\mathcal{M}}$.

(P.T.O)

3. (a) Prove: If $e \in E(G)$ is not a loop, then $\tau(G) = \tau(G - e) + \tau(G \bullet e)$. [6]

(b) Find $\tau(G)$ using Matrix-Tree theorem, for $G = K_{2,3}$. [6]

OR

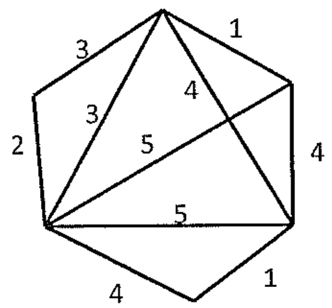
(b) How many trees are there with degree sequence (1,3,1,2,1)? Construct any one such tree. [6]

4. (a) Let f be a flow on a network N = (V, A) with value d. Prove that, if $A(X, \overline{X})$ is a cut in N, then $d = f(X, \overline{X}) - f(\overline{X}, X)$. [6]

(b) Define source, sink and flow in a network N and illustrate these concepts by giving one example of a network N with at least five vertices. [6]

OR

(b) Using Kruscal's algorithm, find a shortest spanning tree for the graph below: [6]



5. (a) Prove: For any graph G, $\chi(G) \leq 1 + \lambda_{\max}(G)$. [6]

(b) (i) Prove: If J is a linear combination of powers of A(G), then G is regular. [6]

(ii) Find sp(($K_{1,3}$).

OR

(b) (i) Prove: If G is bipartite graph, then eigen values of G occur in pair $(\lambda, -\lambda)$ $(\lambda \neq 0)$. [6]

(ii) Give an example of a simple graph G with $\delta(G) < \lambda_{max}(G) = \Delta(G)$.

6. (a) Prove: $R(p, q) \leq R(p-1, q) + R(p, q-1)$, $\forall$ p, q > 2. [6]

(b) Prove: In a hereditary system,
Uniformity property (U) $\Rightarrow$ Sub modularity property (R) [6]

OR

(b) With usual notations, prove that $r(X) \leq r(X + e) \leq r(X) + 1$, for $X \subset E$ and $e \in E$. [6]

x-x-x-x-x-x

SC

(958 A-34) SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester III) Examination

Date: 11-11-2017, Saturday **Time: 10.00 To 1.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH25 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) For $G = K_n$, if diam(G) = D and rad(G) = R, then
(a) D < R (b) D > R (c) D = R (d) D = 2R

(2) A symmetric digraph is
(a) Euler (b) balanced (c) connected (d) regular

(3) Let T be a spanning in-tree with root R. Then
(a) $d^-(R) > 0$ (b) $d^+(R) > 0$ (c) $d^-(R) = 0$ (d) none of these

(4) The order of an incidence matrix of a digraph P_n is
(a) n x n (b) (n – 1) x n (c) n x (n – 1) (d) (n–1) x (n–1)

(5) The coefficient c_6 in chromatic polynomial of $K_{3,3}$ is
(a) 6 (b) 6! (c) 9 (d) 3

(6) Which of the following graphs is not uniquely colorable? (n > 2)
(a) C_n (b) K_n (c) $K_{n,n}$ (d) P_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) maximum ⇒ perfect (c) maximal ⇒ maximum
(b) maximum ⇒ maximal (d) maximal ⇒ perfect

(8) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$
(a) e (b) e^2 (c) ne (d) 2e

2. Attempt any SEVEN: [14]

(a) Prove: If $K_{m,n} = K_{m+n}$, then m = n = 1.
(b) Find $|E(G)|$, if G is a complete, symmetric digraph with n vertices.
(c) Define fundamental circuit matrix in a connected digraph G.
(d) Define spanning out-tree and give one example of it.
(e) Prove: If $\chi(G) = 2$, then G is a bipartite graph.
(f) Prove or disprove: The graph K_n is Hamiltonian, for every n > 2.
(g) Define isomorphic graphs and give one example of it.
(h) Define $\beta(G)$ and $\beta'(G)$ for any graph G.
(i) Prove: The graph C_{2n} has a perfect matching, for every n > 1.

(P.T.O.)

3. (a) Prove that if G is a connected Euler digraph, then it is balanced. [6]

(b) Define symmetric and complete symmetric digraph and give one example of each. Also, discuss the relation between them. [6]

OR

(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

(b) Prove that for each $n \geq 1$ there is a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$, for each $i = 1, 2, \ldots, n$. [6]

OR

(b) Prove: Determinant of every square sub matrix of the incidence matrix of a digraph is $1, -1$ or 0. [6]

5. (a) Prove: A connected graph G is 2-chromatic if and only if it does not contain an odd cycle. [6]

(b) Let G be a planer graph with $n = |V(G)| \geq 6$. Prove that it can be properly colored by at most five colors. [6]

OR

(b) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

6. (a) Let G be a bipartite graph. Prove that $\alpha'(G) = \beta(G)$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Find $\alpha(G)$ and $\beta(G)$ with the corresponding sets, for $G = K_n$. [6]

x-x-x-x-x-x

—x—