SEAT No.________

[125/A-59]

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester III
Tuesday, 11 April 2017
2.00 p.m. to 5.00 p.m.
PS03CMTH01 - Real Analysis II

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) Let $\mathbb{B}$ is the collection of all Borel subsets of $\mathbb{R}$, and let $\mathscr{M}$ be the collection of all measurable subsets of $\mathbb{R}$. Which of the following is true?

(a) $\mathscr{M} \subset \mathbb{B}$ (b) $\mathscr{M} \supset \mathbb{B}$ (c) $\mathscr{M} = \mathbb{B}$ (d) none of these

(2) The Lebesgue measure on $\mathbb{R}$ fails to be .... measure.

(a) finite (b) σ-finite (c) complete (d) saturated

(3) If δ_0 is the point mass measure at 0, then $\delta_0(\mathbb{Q} \cap [-1, 1]) = \ldots\ldots\ldots$

(a) 0 (b) 1 (c) 2 (d) ∞

(4) Let ν be a signed measure and μ be a measure on $(X, \mathscr{A})$. Which of the following implies that ν is a measure?

(a) $\nu \ll \mu$ (b) $\nu \perp \mu$ (c) $\mu = 0$ (d) $\nu(E) \geq 0$, $E \in \mathscr{A}$

(5) Let $f, g \in L^2(\mu)$. Then $f + g$ is in

(a) $L^1(\mu)$ (b) $L^2(\mu)$ (c) $L^\infty(\mu)$ (d) none of these

(6) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Which of the following is not true?
(a) If f is bounded, then f is essentially bounded
(b) If f is essentially bounded, then f is bounded
(c) f is bounded if and only if f is essentially bounded
(d) None of these

(7) If μ^* is an outer measure on X, $E, F \subset X$ and $E \cap F = \emptyset$, then which of the following is true?

(a) $\mu^*(E \cup F) = \mu^*(E) + \mu^*(F)$
(b) $\mu^*(E \cup F) \leq \mu^*(E) + \mu^*(F)$
(c) $\mu^*(E \cup F) \geq \mu^*(E) + \mu^*(F)$
(d) $\mu^*(E \cup F) < \mu^*(E) + \mu^*(F)$

(8) Let $\mathscr{A}_1$ and $\mathscr{A}_2$ be algebras on X. Which of the following is an algebra on X?

(a) $\mathscr{A}_1 \cap \mathscr{A}_2$ (b) $\mathscr{A}_1 \cup \mathscr{A}_2$ (c) $\mathscr{A}_1 - \mathscr{A}_2$ (d) none of these

Q.2 Attempt any *Seven*. [14]

(a) Show that finite measure space is σ-finite measure space.
(b) State Lebesgue Dominated Convergence Theorem.
(c) Show that countable union of positive sets is a positive set.

(d) If ν_1, ν_2 and μ are σ- finite measures on a measurable space $(X, \mathscr{A})$ and both ν_1 and ν_2 are absolutely continuous with respect to μ, then show that $[\frac{d(\nu_1+\nu_2)}{d\mu}] = [\frac{d\nu_1}{d\mu}] + [\frac{d\nu_2}{d\mu}]$.

(e) If f is essentially bounded on a measure space $(X, \mathscr{A}, \mu)$, then show that $|f(t)| \leq \|f\|_\infty$ a.e.

(f) If $(X, \mathscr{A}, \mu)$ is a finite measure space, then show that $L^\infty(\mu) \subset L^1(\mu)$.

(g) State Riesz Representation Theorem.

(h) Let μ^* be an outer measure on X. If $E \subset X$ with $\mu^*(E) = 0$, then show that E is measurable.

(i) If F is a cumulative distribution of a Baire measure μ, then show that F is bounded and increasing.

Q.3

(a) Define integral of a nonnegative measurable simple function. If s and t be nonnegative measurable simple functions on a measure space $(X, \mathscr{A}, \mu)$, then show that $\int_X (s+t)d\mu = \int_X s d\mu + \int_X t d\mu$.

(b) Let f be a measurable function on a measure space $(X, \mathscr{A}, \mu)$. Show that $\int_E f d\mu = 0$ for all $E \in \mathscr{A}$ if and only if $f = 0$ a.e. $[\mu]$ on X.

OR

(b) Let $(X, \mathscr{A}, \mu)$ be a measure space, and let $\{E_n\}$ be a sequence of pairwise disjoint measurable subsets of X. If f is integrable over X, then show that $\int_X f d\mu = \sum_n \int_{E_n} f d\mu$.

Q.4

(c) Let ν be a signed measure on a measurable space $(X, \mathscr{A})$. If $E \in \mathscr{A}$ and $0 < \nu(E) < \infty$, then show that E contains a positive set A with $\nu(A) > 0$.

(d) If ν and μ are σ- finite measures on a measurable space $(X, \mathscr{A})$, then show that there exists a pair of measures ν_0 and ν_1 such that $\nu_0 \perp \mu$, $\nu_1 \ll \mu$ and $\nu = \nu_1 + \nu_2$.

OR

(d) If ν_1 and ν_2 are finite signed measures on a measurable space $(X, \mathscr{A})$, $\alpha, \beta \in \mathbb{R}$ and μ is a measure on $(X, \mathscr{A})$, then show that (1) $|\nu_1 + \nu_2| \leq |\nu_1| + |\nu_2|$ and (2) If $\nu_1 \perp \mu$ and $\nu_2 \perp \mu$, then $(\alpha\nu_1 + \beta\nu_2) \perp \mu$.

Q.5

(e) Show that $(L^p(\mu), \|\cdot\|_p)$ is complete for $1 \leq p < \infty$.

(f) Let $1 < p < \infty$ and $q \in \mathbb{R}$ such that $\frac{1}{p} + \frac{1}{q} = 1$. If $f \in L^p(\mu)$ and $g \in L^q(\mu)$, then show that $fg \in L^1(\mu)$.

OR

(f) Let $1 \leq p < \infty$. Let $f \in L^p(\mu)$, and let $\epsilon > 0$. Show that there is a measurable simple function φ vanishing outside a set of finite measure such that $\|f - \varphi\|_p < \epsilon$.

Q.6

(g) Let μ^* be an outer measure on X. Let $\mathbb{B}$ be the σ- algebra of measurable subsets of X. Define $\overline{\mu} : \mathbb{B} \to [0, \infty]$ by $\overline{\mu}(E) = \mu^*(E)$ for every $E \in \mathbb{B}$. Show that $\overline{\mu}$ is a complete measure on $\mathbb{B}$.

(h) Let μ be a σ- finite measure on an algebra $\mathcal{A}$ of subsets of X, and let μ^* be the outer measure induced by μ. Show that a subset E of X is (μ^*)- measurable if and only if E can be expressed as a difference $E = A - B$, where A is an $\mathcal{A}_{\sigma\delta}$- set and $\mu^*(B) = 0$.

OR

(h) Let μ be a measure on an algebra $\mathcal{A}$ of subsets of X, and let μ^* be the outer measure induced by μ. Prove that $\mu^* = \mu$ on $\mathcal{A}$ and every member of $\mathcal{A}$ is measurable.

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SEAT No.________

[ ]

[122/A65]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - III Examination

Thursday, 13th April, 2017

PS03CMTH02, Mathematical Methods I

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: All the questions are to be answered in answer book only. Figures to the right indicate full marks of the respective question. Assume standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions: [8]

1. The set $\{\sin nx : n \in \mathbb{N}\}$ on the interval $[-\pi, \pi]$ is ____.
   (a) orthogonal but not orthonormal (b) orthonormal (c) functions of zero norm (d) trivial
2. Let $f(x) = x - x^2$, $-1 \le x < 1$ be 2-periodic. Then the Fourier series of f at $x = 1$ converges to ____.
   (a) -2 (b) 0 (c) -1 (d) 1
3. If $f \in L^1(\mathbb{R})$ is an odd function, then ____.
   (a) $F[f] = iF_s[f]$ (b) $F[f] = -iF_s[f]$ (c) $F[f] = F_c[f]$ (d) $F[f] = -F_c[f]$
4. If $F(s)$ denotes Fourier transform of $f(x)$, then Fourier transform of $f(5x)$ is ____.
   (a) $F(5s)$ (b) $\frac{1}{5}F\left(\frac{s}{5}\right)$ (c) $\frac{1}{5}F(5s)$ (d) $5F\left(\frac{s}{5}\right)$
5. $L[e^{2t}\cosh 3t](t) =$ ____.
   (a) $\frac{2}{(s-2)^2-9}$ (b) $\frac{3}{(s-2)^2-9}$ (c) $\frac{s-2}{(s-2)^2-9}$ (d) $\frac{s-3}{(s-2)^2-9}$
6. $L^{-1}\left[\frac{L[f](s)}{s}\right](t) =$ ____.
   (a) $\int_t^\infty L[f](u)du$ (b) $\int_0^t L[f](u)du$ (c) $\int_t^\infty f(u)du$ (d) $\int_0^t f(u)du$
7. The Z-transform of $(\cos n\pi)_{n\ge 0}$ is ____.
   (a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{1}{z+1}$ (d) $\frac{1}{z-1}$
8. The domain of convergence of Z-transform of $(3^n)_{n\ge 0}$ is ____.
   (a) $|z| < \frac{1}{3}$ (b) $|z| > \frac{1}{3}$ (c) $|z| < 3$ (d) $|z| > 3$

Q-2 Attempt *Any Seven* of the following: [14]

(a) State Dirichlet theorem for Fourier series.

(b) Compute the half range Fourier cosine series of $f(x) = \frac{1}{2}$, $0 < x < \pi$.

(c) Compute the Fourier transform of $xe^{-\frac{x^2}{2}}$ provided that $F\left[e^{-\frac{x^2}{2}}\right](s) = e^{-\frac{s^2}{2}}$.

(d) State and prove Parseval's identity for Fourier transform.

(e) Let $u(x,t)$ be a function of two variables such that both $u(x,t)$ and $u_x(x,t)$ tend to 0 as $x \to \infty$, then show that $F_s[u_{xx}](s) = \sqrt{\frac{2}{\pi}}u_x(0,t) - s^2F_s[u](s)$; notation being usual.

(f) In usual notations, show that $L[t^n f(t)](s) = (-1)^n \frac{d^n}{ds^n} L[f](s)$.

(g) Compute the inverse Laplace transform of $\log\left(\frac{s^2+4}{s^2+9}\right)$.

(h) Find $H_2(x)$ and hence evaluate $H_2(1)$; notation being usual.

(i) Compute the Z-transform of $(\sin(\alpha n))_{n\geq 0}$.

Q-3 (a) Compute the Fourier series of a 2π-period function $f(x) = x + x^2$, $-\pi \leq x < \pi$. Hence find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. [6]

(b) Compute half range Fourier sine series of the function $f(x) = \pi x - x^2$, $0 < x < \pi$. Use Parseval's identity to evaluate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$. [6]

OR

(b) Solve $u_t = c^2 u_{xx}$, $0 \leq x \leq 100$, $t > 0$ subject to $u(0,t) = u(100,t) = 0$, for all t and $u(x,0) = \begin{cases} x & , \quad 0 \leq x \leq 50 \\ 100 - x & , \quad 50 \leq x \leq 100. \end{cases}$ [6]

Q-4 (a) Compute the Fourier transform of e^{-ax^2}, $a > 0$. [6]

(b) Using Fourier transform methods, solve $u_t = k u_{xx}$, $x > 0$, $t > 0$ subject to $u(x,0) = 0$ for all $x > 0$, $u_x(0,t) = -a$ for all $t > 0$, and $u, u_x \to 0$ as $x \to \infty$. [6]

OR

(b) Let $f, g \in L^1(\mathbb{R})$. Define convolution product of f and g. Show that $f * g \in L^1(\mathbb{R})$ and $F[f * g] = F[f]F[g]$. [6]

Q-5 (a) Using methods of Laplace tranform, solve $u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$ subject to $u(0,t) = u(1,t) = 0$ for all t, $u(x,0) = \sin \pi x$ and $u_t(x,0) = -\sin \pi x$ for all x. [6]

(b) Compute the inverse Laplace transform of the functions $\dfrac{s}{(s^2+4)^2}$ and $\dfrac{1}{(s-1)(s^2+1)}$. [6]

OR

(b) Use Laplace transform methods to solve $y'' + 6y' + 9y = 6t^2 e^{-3t}$ subject to $y(0) = 1$, $y'(0) = 2$. [6]

Q-6 (a) State Gram-Schmidt orthonormalization process and hence orthonormalize the set $\{1, x, x^2\}$ over $[-1, 1]$. [6]

(b) i. Find the Green's function for $y'' = f(x)$ subject to $y(0) = 0 = y(1)$ and hence find its solution when $f(x) = x^2$. [3]

ii. Show that $H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0$ for all $n \geq 1$. [3]

OR

(b) i. Find a polynomial of degree 2 so that $\int_{-1}^{1} |\sin x - p(x)|^2\, dx$ is minimum. [3]

ii. Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0$ subject to $y_0 = 1$, $y_1 = 2$. [3]

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[93/A-52]

SEAT No.________

No of printed pages: **2**

Sardar Patel University

Mathematics

M.Sc. Semester III

Monday, 17 April 2017

2.00 p.m. to 5.00 p.m.

PS03EMTH01 - Functional Analysis II

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) Let $1 \leq p < q \leq \infty$. Which of the following is true?

(a) $\ell^p \subset \ell^q$ (b) $\ell^p \supset \ell^q$ (c) $\ell^p = \ell^q$ (d) none of these

(2) Let E be a subset of a normed space X. Which of the following is not true?
- (a) If E is convex, then E^0 is convex.
- (b) If E is convex, then $\overline{E}$ is convex.
- (c) If E is convex, then $E + x$ is convex for all $x \in X$.
- (d) If E is convex, then $\overline{E} = \overline{E^0}$

(3) Let $\{0\} \neq X$ and Y be normed spaces, and let $T : X \to Y$ be linear and continuous. Let $\alpha = \sup\{\|Tx\| : \|x\| < 1\}$ and $\beta = \sup\{\|Tx\| : \|x\| = 1\}$. Which of the following is true?

(a) $\alpha = \beta$ (b) $\alpha < \beta$ (c) $\alpha > \beta$ (d) none of these

(4) The dimension of a hyperspace in the vector space of all $n \times n$ matrices over $\mathbb{R}$ is ......

(a) n^2 (b) $\frac{n(n+1)}{2}$ (c) $\frac{n(n-1)}{2}$ (d) $n^2 - 1$

(5) Which of the following is not a Banach space with the sup norm?

(a) c_0 (b) $C[0,1]$ (c) $\mathbb{P}[0,1]$ (d) $C_0(\mathbb{R})$

(6) Let $D_m(x) = \sum_{k=-m}^{m} e^{ikx}$. Then the value of $\lim_{n\to\infty} \|D_n\|_1$ is ......

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) ∞

(7) Let $F \in BL(X,Y)$ and $G \in BL(Y,Z)$. Which of the following is not true?

(a) $\|F\| = \|F'\|$ (b) $\|F\| = \|F''\|$ (c) $(G \circ F)' = G' \circ F'$ (d) $(G \circ F)' = F' \circ G'$

(8) The dual of ℓ^2 is isometrically isomorphic to

(a) ℓ^1 (b) c_0 (c) ℓ^∞ (d) ℓ^2

Q.2 Attempt any *Seven*. [14]

(a) Let Y be a closed subspace of a normed space X. If x and y are in X, then show that $|||x + y + Y||| \leq |||x + Y||| + |||y + Y|||$

(b) Show that $\|\cdot\|_1$ and $\|\cdot\|_\infty$ are equivalent on $\mathbb{K}^n$.

(c) Let X and Y be normed spaces, and let $F : X \to Y$ be linear. If $\|F(x)\| \leq 2\|x\|$ for all $x \in X$, then show that F is uniformly continuous.

(d) Let $f : (\ell^1, \|\cdot\|_1) \to \mathbb{K}$ be $f((x(k))) = \sum_{k=1}^{\infty} x(k)$. Find the norm of f.

(e) Let X be a normed space. Show that $\|a\| = \sup\{|f(a)| : f \in X', \|f\| \leq 1\}$ for all $a \in X$.

(f) Let X and Y be normed spaces, and $F : X \to Y$ be linear. If F is an open map, then show that F is surjective.

(g) Let P be a projection on a normed space X. If both $Z(P)$ and $R(P)$ are closed in X, then show that P is closed.
(h) Define weak*- convergence of a sequence. Show that weak limit of a sequence is unique.
(i) Let X be a normed space, and let $T \in BL(X)$ be invertible. Show that T is bounded below.

Q.3

(a) Let (a_n) and (b_n) be sequences in $\mathbb{K}$. Let $1 < p < \infty$ and q be such that $\frac{1}{p} + \frac{1}{q} = 1$. Show that $\sum_n |a_n b_n| \leq (\sum_n |a_n|^p)^{\frac{1}{p}} (\sum_n |b_n|^q)^{\frac{1}{q}}$. State the result you use. [6]

(b) Let X and Y be a normed spaces, and let X be finite dimensional. Show that every linear map from X to Y is continuous. Is the same true if X is infinite dimensional? Justify. [6]

OR

(b) Let $T : (\mathbb{K}^n, \|\cdot\|_1) \to (\mathbb{K}^m, \|\cdot\|_1)$ be linear, and let (α_{ij}) be the matrix of T. Find the norm of T. Do the same when we consider $\mathbb{K}^n$ and $\mathbb{K}^m$ with the norm $\|\cdot\|_\infty$. [6]

Q.4

(c) State and prove Hahn-Banach Extension Theorem. [6]

(d) When is a series $\sum_n x_n$ called summable in a normed space X? When is it called absolutely summable? Let X be a normed space. Show that X is a Banach space if and only if every absolutely summable series of elements of X is summable. [6]

OR

(d) Let X be a normed space. Show that X can be embedded in its second dual. Also, show that if Z and Z' are completions of X, then Z and Z' are isometrically isomorphic. [6]

Q.5

(e) State and prove Closed Graph Theorem. [6]

(f) Let $\|\cdot\|$ be a complete norm on $L^1[-\pi, \pi]$ such that if a sequence (x_n) in $L^1[-\pi, \pi]$ converges to $x \in L^1[-\pi, \pi]$, then $\widehat{x_n}(j) \to \widehat{x}(j)$ for all $j \in \mathbb{Z}$. Show that $\|\cdot\|$ is equivalent to $\|\cdot\|_1$. State results you use. [6]

OR

(f) Show that the statement of Open Mapping Theorem is not true if either X is not a Banach space or Y is not a Banach space. [6]

Q.6

(g) If $1 \leq p < \infty$, then show that ℓ^p is separable. Show that ℓ^∞ is not separable. [6]

(h) Let X be a normed space, and let $T \in BL(X)$ be of finite rank. Show that $\sigma_e(T) = \sigma_a(T) = \sigma(T)$. [6]

OR

(h) (क) Let $T \in BL(X, Y)$. Show that T is not bounded below if and only if there is a sequence (x_n) in X such that $\|x_n\| = 1$ for all n and $\|Tx_n\| \to 0$ as $n \to \infty$. [3]

(ख) Let X be a Banach space, and $T \in BL(X)$. If $\|T\| < 1$, then show that $I - T$ is invertible. [3]

SEAT No.______ No of printed pages: 2

[62/A-36]

Sardar Patel University

M.Sc. Semester III Examination

Wednesday, 19^{th} April 2017; 14:00 to 17:00

Subject: Mathematics; Code: PS03EMTH02; Title of Paper: Banach Algebras

Maximum Marks: 70

Q.1 To answer, write the correct **question number and option number only.** [8]

(*a*) In a Banach algebra $\mathcal{A}$ with identity $\mathbf{1}$, $\|\mathbf{1}\|$ ____ 1.

(i) $=$ (ii) $>$ (iii) $\leq$ (iv) $\geq$

(*b*) ____ is not a Banach algebra.

(i) $P[0,1]$ (ii) ℓ^∞ (iii) $\ell^1(\mathbb{Z}_5)$ (iv) $C[0,1]$

(*c*) Spectrum of $f \in C[0,1]$, defined by $f(x) = x^2 + 1$, $(x \in [0,1])$, is ____.

(i) $[0,1]$ (ii) $[1,2]$ (iii) $\{0\}$ (iv) $\{1\}$

(*d*) Spectral radius of $f \in C[0, 7.987]$, defined by $f(x) = \sin x$, $(x \in [0, 7.987])$, is ____.

(i) 1 (ii) -1 (iii) π (iv) $-\pi$

(*e*) The Banach algebra ____ with usual multiplication and norm is simple.

(i) $BL(\ell^2)$ (ii) $C[0,1]$ (iii) $\mathbb{C}^7$ (iv) $BL(\mathbb{C}^6)$

(*f*) For a normal element x of a C^*-algebra, ____.

(i) $r(x) \leq 1$ (ii) $x^* = x$ (iii) $\|x\| = r(x)$ (iv) $1 + x \in G$

(*g*) The Gel'fand transform of ____ is an isometry.

(i) $C^1[0,1]$ (ii) ℓ^1 (iii) ℓ^2 (iv) ℓ^∞

(*h*) For a/an ____ element x of a C^*-algebra, the $\text{sp}(x)$ is real.

(i) normal (ii) selfadjoint (iii) invertible (iv) singular

Q.2 Attempt any **Seven. (Start a new page.)** [14]

(*a*) Prove that multiplication is continuous on a normed algebra.

(*b*) Prove or disprove: The sequence $\{f_n\}$, where $f_n(t) = t^n$, $(t \in [0,1])$, $n \in \mathbb{N}$ is a Cauchy sequence in $C[0,1]$.

(*c*) Define *spectrum of an element of an algebra.* Find spectrum of $\begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix} \in M_2(\mathbb{C})$.

(*d*) Show that spectrum of an element of a Banach algebra is closed.

(*e*) Mention one maximal ideal of $\mathbb{C}^3$ with usual multiplication.

(*f*) For an element x of a Banach algebra $\mathcal{A}$, prove that $r(x) \leq \|x\|$.

(*g*) In usual notations, prove that $\frac{x(\lambda) - x(\mu)}{\lambda - \mu} = x(\lambda)x(\mu)$.

(*h*) Show that $\{f \in C^1[0,1] : f(0.1) = f'(0.1) = 0\}$ is a closed ideal of $C^1[0,1]$ which is not maximal.

(*i*) Show that $\psi : C[0,1] \to C[1,2]$ defined by $\psi(f)(x) = f(x-1)$, $(f \in C[0,1])$, is an algebra homomorphism.

[Contd...]

Q.3 **(Start a new page.)**

(a) Define *Banach algebra* and show that for a compact T_2-space X, $(C(X), \|\cdot\|_\infty)$ is a Banach algebra. [6]

(b) In usual notations, prove that $Z \subset S$ and $\text{bd}(S) \subset Z$. [6]

OR

(b) Show that the set of all invertible elements of a Banach algebra is open. [6]

Q.4 **(Start a new page.)**

(c) Obtain the spectral radius formula in a complex unital Banach algebra. [6]

(d) Show that a complex homomorphism on a Banach algebra is automatically continuous. Also show that two complex homomorphisms cannot have the same kernel. [6]

OR

(d) Show that every closed proper ideal of a commutative unital Banach algebra is contained in a maximal ideal. [6]

Q.5 **(Start a new page.)**

(e) For a Banach algebra $\mathcal{A}$, show that the Gel'fand transform $x \in \mathcal{A} \mapsto \widehat{x} \in C(m(A))$ is a nórm-decreasing homomorphism. [6]

(f) Giving all details, show that radical of a commutative unital complex Banach algebra $\mathcal{A}$ is a two sided closed ideal of $\mathcal{A}$. [6]

OR

(f) Let $\mathcal{A}$ and $\mathcal{B}$ be two isomorphic commutative unital complex Banach algebras. Show that their Gel'fand spaces are homeomorphic. [6]

Q.6 **(Start a new page.)**

(g) Let $\mathcal{A}$ be a commutative unital complex Banach algebra. Show that the Gel'fand transform $x \in \mathcal{A} \mapsto \widehat{x} \in C(m(A))$ is an isometry if and only if $\|x^2\| = \|x\|^2$ for all $x \in \mathcal{A}$. [6]

(h) State and prove that Gel'fand-Naimark theorem for a commutative unital C^*-algebras. [6]

OR

(h) Define a *Banach* algebra* and show that $A(\mathbb{D})$ is a Banach* algebra which is not a C^*-algebra. [6]

✠‡✠‡✠‡✠†✠

SEAT No.________

[26/A-18]

No of printed pages: 2

Sardar Patel University

Mathematics

M.Sc. Semester III

Friday, 21 April 2017

2.00 p.m. to 5.00 p.m.

PS03EMTH03 - Problems and Exercises in Mathematics II

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) Let $f(x) = \lim_{n\to\infty} \sin^{2n} x$, $x \in [0, 2]$. Then $\int_0^2 f(x)dx$ equals

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

(2) Let $a_n = \frac{1}{n^2}$, $b_n = \frac{1}{2}(3 + \frac{1}{n})$ and $c_n = a_n + b_n$. Then $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} c_k = \ldots\ldots$

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 3

(3) The residue of $f(z) = \frac{\sinh z}{z}$ at 0 is

(a) 1 (b) $2\pi i$ (c) -1 (d) 0

(4) Let f be an entire function satisfying $|f(z)| \leq \frac{2}{1+|z|^2}$ for all $z \in \mathbb{C}$. Then $f(0) = \ldots\ldots$

(a) 0 (b) 1 (c) 2 (d) none of these

(5) Let G and G' be finite groups, and let $f : G \to G'$ be a homomorphism. Which of the following is true?

(a) $o(a) = o(f(a))$ (b) $o(a) \mid o(f(a))$ (c) $o(f(a)) \mid o(a)$ (d) none of these

(6) Which of the following is a conjugate to $(1\ 2)(3\ 5) \in S_5$?

(a) $(2\ 3)(4\ 5)$ (b) $(1\ 2\ 3)$ (c) $(1\ 2\ 3\ 5)$ (d) $(5\ 3\ 2\ 1)$

(7) Let τ be the topology on $\mathbb{R}$ generated by a basis $\mathcal{B} = \{[a, b] : a, b \in \mathbb{R}, a \leq b\}$, and let σ be the standard topology on $\mathbb{R}$. Which of the following is true?

(a) $\tau = \sigma$ (b) $\tau \subset \sigma$ (c) $\tau \supset \sigma$ (d) none of these

(8) The number of onto functions from $X = \{1, 2, \ldots, n\}$ to itself is ......

(a) n (b) 2^n (c) n^2 (d) $n!$

Q.2 Attempt any *Seven*. [14]

(a) Show that the sequence $f_n(x) = \sin^n x \cos^n x$ converges uniformly on $\mathbb{R}$.

(b) Evaluate $\int_{[0,1]\times[0,1]} [x + y] dxdy$.

(c) If f is analytic on a domain D and if $|f|$ is a constant map, then show that f is a constant map.

(d) Let f and g be analytic functions on a domain D, and let $n \in \mathbb{N}$. If $f^n = g^n$, then show that $f = \alpha g$ for some $\alpha \in \mathbb{C}$ with $|\alpha| = 1$.

(e) Let X be a topological space. If $\{(x, x) : x \in X\}$ is closed in $X \times X$, then show that X is Hausdorff.

(f) If (x_n) is a Cauchy sequence in a discrete metric space, then show that (x_n) is eventually constant.
(g) Let $O_n(\mathbb{R})$ be the collection of all orthogonal $n \times n$ real matrices. Show that $O_n(\mathbb{R})$ is disconnected.
(h) Determine all group automorphisms on $\mathbb{Z}$.
(i) Let H and K be subgroups of a finite group G. If $(o(H), o(K)) = 1$, then show that $H \cap K = \{e\}$.

Q.3
(a) Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function satisfying $f(x^2) = f(x)$ for all x. Show that f is a constant map. Is the same true if f is not continuous? Why? [6]
(b) Let $A \subset [a, b]$. Show that χ_A is Riemann integrable over $[a, b]$ if and only if the measure of the boundary of A is 0. State the result you use. [6]

OR

(b) Let $f : (a, b) \to \mathbb{R}$ be continuous. Show that f is uniformly continuous if and only if f can be extended as a continuous function on $[a, b]$. [6]

Q.4
(c) Let f be an entire function satisfying $|f(z)| \leq 5 + 7|z|^8$ for all $z \in \mathbb{C}$. Show that f is a polynomial of degree at most 8. State the results you use.
(d) Let $u : \mathbb{R}^2 \to \mathbb{R}$ be a harmonic function. If either u is bounded above or u is bounded below, then show that u is a constant map. [6]

OR

(e) Let f be analytic at z_0, and let z_0 be a zero of f. When is z_0 called a zero of f of order n? Suppose that f and g are analytic at z_0. If z_0 is a zero of f of order 2 and z_0 is a zero of g of order 3, then show that z_0 is a simple pole of $\frac{f}{g}$. Also, find the residue of $\frac{f}{g}$ at z_0. [6]

Q.5
(f) Let G be a group of order pq, where p and q are primes such that $p < q$ and $p \nmid q - 1$. Prove that G is cyclic. State carefully results you use. [6]
(f) Let a and b be elements of a finite group G. If $ab = ba$ and $(o(a), o(b)) = 1$, then show that $o(ab) = o(a)o(b)$. [6]

OR

(g) (क) Show that there is no group G for which $o(G/Z(G)) = 77$. [3]
(ख) Write all abelian groups (up to isomorphism) of order 120. [3]

Q.6
(h) Let X and Y be topological space, and let Y be Hausdorff. If $f : X \to Y$ is continuous, then show that $\{(x, f(x)) : x \in X\}$ is a closed subset of $X \times Y$. Is the converse true? Why? [6]
(i) Define basis for a topology on a set X. Show that the topology on X generated by a basis $\mathcal{B}$ is the smallest topology containing $\mathcal{B}$. [6]

OR

(h) Let (X, d) be a metric space, and let A be a nonempty subset of X. Let $x \in X$. Show that $x \in \overline{A}$ if and only if $d(x, A) = 0$. [6]

[27/A-19]

SEAT No._______

No of printed pages: 2

Sardar Patel University

M.Sc. Semester III Examination

Friday, 21^{st} April 2017; 14:00 to 17:00

Subject: Mathematics; Code: PS03EMTH08; Title of Paper: Group Theory

Maximum Marks: 70

Note: Notations are standard.

Q.1 To answer, write the correct question number and option number only. [8]

(a) For an element a of a group G, $a^{m+n} =$ _____.

(i) a^{mn} (ii) $(a^m)^n$ (iii) $a^n a^m$ (iv) none of the other three

(b) The inverse of the permutation (1 2 3) is _____.

(i) (1 2 3) (ii) (2 3 1) (iii) $(1\ \frac{1}{2}\ \frac{1}{3})$ (iv) (2 1 3)

(c) For a finite abelian subgroups H, K of a group G, $o(HK)o(H \cap K) =$ _____.

(i) $o(H)o(K)$ (ii) $o(H)/o(K)$ (iii) $o(H) + o(K)$ (iv) $o(HK)$

(d) In S_3 the number of elements in the conjugacy class $C(2\ 3)$ is _____.

(i) 1 (ii) 2 (iii) 3 (iv) 6

(e) Total number of conjugacy classes in S_4 is _____.

(i) 2 (ii) 3 (iii) 4 (iv) 5

(f) A group of order _____ is simple.

(i) 3 (ii) 4 (iii) 6 (iv) 8

(g) A group of order _____ is abelian.

(i) 6 (ii) 8 (iii) 16 (iv) 25

(h) There are _____ nonisomorphic finite abelian groups of order 16.

(i) 1 (ii) 3 (iii) 5 (iv) 7

Q.2 Attempt any Seven. **(Start a new page.)** [14]

(a) Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, where $a, b, c, d \in \mathbb{Z}$. Show that $ad - bc = \pm 1$.

(b) Let G be a group. Show that every $g \in G$ has a unique inverse in G.

(c) Let G be a group of order 18. Show that there is $a \in G \setminus \{e\}$ such that $a^4 = e$.

(d) Let G be a finite group and $a \in G$. Show that $o(a) \mid o(G)$.

(e) Prove that the relation of conjugacy is a transitive relation on a group.

(f) Give one reason to conclude that a group of order 9 is abelian.

(g) Let A, B be subgroups of a group G. For $x, y \in G$ define $x \sim y$ if there exist $a \in A, b \in B$ such that $y = axb$. Show that $\sim$ is transitive.

(h) Let G be an abelian group and $\widehat{G}$ be the set of all homomorphisms from G to the group of all nonzero complex numbers. Show that $\widehat{G}$ is abelian.

(i) For $n \in \mathbb{N}$ define $\varphi : S_n \to \mathbb{Z}_2$ by $\varphi(\theta) = \begin{cases} 0 & \text{if } \theta \text{ is even} \\ 1 & \text{if } \theta \text{ is odd.} \end{cases}$ Show that φ is a homomorphism.

[Contd...]

Q.3 **(Start a new page.)**

(a) Define a *group* and show that the set S_3 of all permutations on three symbols is a nonabelian group. [6]

(b) Let H, K be two subgroups of a group G. Show that HK is a subgroup of G if and only if $HK = KH$. [6]

OR

(b) State and prove Cayley's Theorem [6]

Q.4 **(Start a new page.)**

(c) Define *automorphism* of a group. If G is a group, then prove that the set of all automorphisms of G is also a group. [6]

(d) Let G be a finite group and $a \in G$. In usual notations, prove that $c_a = \text{o}(G)/\text{o}(N(a))$. [6]

OR

(d) If G is a group with $\text{o}(G) = p^n$ for some prime p and $n \in \mathbb{N}$, then show that $Z(G) \neq \{e\}$. [6]

Q.5 **(Start a new page.)**

(e) Let G be a finite group and $p \in \mathbb{N}$ be prime such that $p^n \mid \text{o}(G)$. Show that G has a subgroup of order p^n. [6]

(f) Let $p \in \mathbb{N}$ be prime. Let $n(k) \in \mathbb{N}$ be such that $p^{n(k)} \mid (p^k)!$ but $p^{n(k)+1} \nmid (p^k)!$. Show that [6]

$$n(k) = 1 + p + p^2 + \cdots + p^{k-1}.$$

OR

(f) Define *solvable group*. Show that S_5 is not solvable. [6]

Q.6 **(Start a new page.)**

(g) Define *internal direct product*. Let G be an internal direct product of its subgroups $N_1, N_2, \ldots, N_k$ and let $1 \leq i < j \leq k$. Show that $N_i \cap N_j = \{e\}$. Also for $a \in N_i$, $b \in N_j$, show that $ab = ba$. [6]

(h) For to isomorphic abelian groups G and G', and $s \in \mathbb{N}$, show that $G(s)$ is isomorphic to $G'(s)$, where $G(s) = \{x \in G : x^s = e\}$. [6]

OR

(h) Let $p \in \mathbb{N}$ be a prime. Prove that the number of nonisomorphic abelian groups of order p^n is equal to the number of partitions of n. [6]

✠‡✠‡✠‡✠†✠

SEAT No.________

[63/A-37]

No of printed pages: 4

Sardar Patel University

M.Sc. (Sem-III), PS03EMTH12, Financial Mathematics-I;
Wednesday, 19th April, 2017; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) The required table of normal distribution is attached with this question paper; (ii) Calculator is allowed.

Q.1 Answer the following. [8]

1. How many types of basic financial derivatives are there?
   (A) 4 (B) 3 (C) 5 (D) 2
2. How many types of participants in options market ?
   (A) 2 (B) 3 (C) 4 (D) 1
3. How many types of repo rates available?
   (A) 3 (B) 2 (C) 4 (D) 1
4. The variances of the change in successive time periods in Markov process are
   (A) additive (B) not additive (C) multiplicative
   (D) not multiplicative
5. The mean change per unit time is known as
   (A) repo rate (B) variance rate (C) drift rate
   (D) none of these
6. The solution of $y' + y = 0$ is
   (A) $e^{-x} + c$ (B) ce^{-x} (C) $\ln x + c$ (D) none of these
7. The BSM differential equation is of (order,degree)
   (A) $(2,1)$ (B) $(2,2)$ (C) $(1,2)$ (D) $(1,1)$
8. The delta for European put option is
   (A) 0 (B) non negative (C) non positive
   (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define derivative and give one example of it.
(b) Explain the difference between buying a call option and selling a put option.
(c) Write full form of LIBOR and LIBID.
(d) Define repo rate.
(e) A stock price is currently \$20. It is known that at the end of 3 months it will be either \$23 or \$17. The risk free interest rate is 12% per annum with continuously compounding. What is the value of a 3 months European call option with a strike price of \$21 ?
(f) Suppose S follows the process $dS = \mu S dt + \sigma S dz$. What is the process followed by $\ln S$?
(g) Give formula of two step binomial model.
(h) Define Unit Impulse function.
(i) Write down BSM formulas for European options on an asset paying no dividend.

Q.3 [12]

(a) Explain arbitrageur with an example.
(b) Define (i) Exchange traded market; (ii) Futures contract; (iii) Speculator

OR

(b) Consider a call option of a share with striking price \$50, option value \$7 per share and it expires after 3 months. Under what circumstances will the seller of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.

Q.4 [12]

(a) Explain Generalized Wiener process.
(b) Prove (i) $R_c = m\ln(1+\frac{R_m}{m})$; (ii) $R_m = m(e^{\frac{R_c}{m}}-1)$.

OR

(b) An investor will receive ₹1200 after one year for an investment of ₹1000 now. Calculate the percentage return per annum with
(i) semi annual compounding; (ii) quarterly compounding; (iii) continuously compounding.

Q.5 [12]

(a) Discuss simple model for stock prices.
(b) Explain generalized one step binomial model.

OR

(b) Find the price of a 6 months European put option with a strike price \$42 on a stock whose current price is \$40. Also the stock price either moves up or down by 10% for each 3 months period and the risk free interest rate is 12% per annum with continuously compounding.

Q.6 [12]

(a) Derive the put-call parity for European options.
(b) Show that $\Delta_C = 1 - N(-d_1)$.

OR

(b) Find the value of an European call option on a non-dividend paying stock when the current stock price is ₹50, the strike price is ₹55, the risk free interest rate is 10% per annum with continuously compounding, the volatility is 40% per annum, and the time to maturity is 3-months.

*****

Table for $N(x)$ When $x \geqslant 0$

This table shows values of $N(x)$ for $x \geqslant 0$. The table should be used with interpolation. For example,

$$N(0.6278) = N(0.62) + 0.78[N(0.63) - N(0.62)]$$
$$= 0.7324 + 0.78 \times (0.7357 - 0.7324)$$
$$= 0.7350$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for $N(x)$ When $x \leqslant 0$

This table shows values of $N(x)$ for $x \leqslant 0$. The table should be used with interpolation. For example,

$$N(-0.1234) = N(-0.12) - 0.34[N(-0.12) - N(-0.13)]$$
$$= 0.4522 - 0.34 \times (0.4522 - 0.4483)$$
$$= 0.4509$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

[94]
[A-53]

SEAT No.________ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-III) Examination
April – 2017
Monday - 17/04/2017
Time: 02:00 PM to 05:00 PM

Subject: Mathematics **Course No. PS03EMTH16(Relativity-I)**

Note: 1. Answer to all questions to be given in the answer book only.
2. Figures on the right indicate full marks.

Q-1 Choose appropriate answer from the options given. (08)

1. In General Galelian transformation space coordinates are ________.
   (a) relative (b) absolute (c) not defined (d) none of these
2. Newton's equations are invariant under __________ transformation.
   (a) Special Lorentz (b) General Lorentz
   (c) Special Galelian (d) None of these
3. A frame moving with uniform acceleration relative to an inertial frame is ______
   (a) an inertial frame (b) a rotating frame
   (c) a non-inertial frame (d) not a frame
4. Volume of an object is________ under Special Lorentz transformations
   (a) Absolute (b) relativistic (c) non-deterministic (d) non-relativistic
5. In Special Relativity, moving clocks appear to run ________
   (a) slow (b) fast (c) with constant speed (d) uniformly
6. Momentum 4-vector is ___________
   (a) time-like (b) of constant magnitude (c) null (d) contravariant
7. Which one of the following is not correct according to Special Relativity?
   (a) Mass is equivalent to energy.
   (b) Mass changes with motion.
   (c) Mass of a particle remains constant during the motion.
   (d) Rest mass and moving mass of a photon are different.
8. The condition for flat space is________.
   (a) $R_{hijk} = 0$ (b) $R_{ij} = 0$ (c) $R = 0$ (d) $g_{ij} = 0$

Q-2 Attempt any **SEVEN** (14)

1. State general Galelian transformation.
2. State postulates of special relativity.
3. What is the formula for relativistic composition of velocities.
4. Explain the meaning of time dilation.
5. Give expression of the spacetime interval.
6. Define a space-like vector.
7. State the expression of transformation of a contravariant vector.
8. What is meant by Minkowski structure of spacetime.
9. State the expressions of Christoffel symbols of both kinds.

Q-3

(a) Show that wave equation is not invariant under special Galilean transformation. (06)

(b) Show that composition of two special Lorentz transformations is a special Lorentz transformation. (06)

OR

(b) A rod of length 50 cm is moving with velocity 0.9c in a direction inclined at 60° to its own length. Find the apparent length of this rod.

Q-4

(a) Discuss the concept of Doppler effect using special relativity. (06)

(b) Show the spacetime interval is invariant under SLT. (06)

OR

(b) A source of red signal (wavelength 7500 A^{o}) moves with velocity 1.5×10^{8} meters per second. What will be the apparent wavelength of the light?

Q-5

(a) Define velocity 4-vector. Show that it has constant norm. (06)

(b) Define momentum 4-vector and find its components. (06)

OR

(b) Mass of moving particle appears to increased by 25% of its rest mass. Find its velocity.

Q-6

(a) Explain the meaning of a covariant vector with example. (06)

(b) State geodesic equation and show that geodesics in a plane are straight lines. (06)

OR

(b) Discuss principle of equivalence.

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SEAT No.________

[7/A-13] No. of Printed Pages : 2

Sardar Patel University

M. Sc. *(Third Semester)* Examination

Monday, April 24, 2017

Course No. PS03E, MTH24 : *C Programming and Mathematical Algorithms II*

Time: 02.00 p.m. to 04.00 p.m. *Maximum marks: 35*

Note: Figures to the right indicates marks.

1. *Choose appropriate answer to the question from the given options.* [5]

i) Suppose ***double p, *q;*** is a type declaration. Then which of the following ***is true***?
(a) &p = q; (b) q = &p; (c) p = q; (d) q = p;

ii) Suppose ***int *u, v***; and &v is 8120. If u = &v + 2;, then the content of u is ___ .
(a) 8124 (b) 8122 (c) 8128 (d) none of these

iii) Which of the following is valid first line of a function?
(a) float f(float &x, int &y) (b) float f(x, int y)
(c) float f(float *x, int *y) (d) float f(float *x, int &y)

iv) Which of the following ***is true***?
(a) Declaration of static and automatic variables in the function main() have same role.
(b) Declaration of static and external variables in the function main() have same role.
(c) Declaration of external and automatic variables in the function main() have same role.
(d) None of these is true.

v) ***int z = 3; void f(int a) {z = z * a} void main(){f(2); f(5);}*** what is the content of z after the execution of the program?
(a) 4 (b) 6 (c) 30 (d) none of these

2. Answer any THREE of the following: [6]

a) Write the meaning of passing by values and passing by references.
b) What is a data file? What are different type of data files.
c) Which operators one can operate on pointer variables.
d) In the graphics mode, write the effect of following functions:
void ***putpixel*** (int x, int y, int c);
void ***line***(int x0, int y0, int x1, int y1);

3. a) (i) What is a static variable? How does it declare? How does it carry value? Explain with example. [3]

(ii) Write note on function definition in C programming. [3]

b) (i) Write an algorithm to find the value of a polynomial p(x) at z, where [2]
$p(x) = a_0 + a_1(x - c_1) + a_2(x - c_1)(x - c_2) + \ldots\ldots + a_n(x - c_1)\ldots\ldots(x - c_n)$.

(ii) If ***float a[10];*** is declared in a program, write the meaning of each of the following: [2]
a, *a, a+3, *(a+3).

(iii) int a, *x; float b, *y; are declarations. State why each of the following is invalid. [2]
x = y y = &a &b = y b = &a

OR

b) (i) Find the values of a and b after the execution of each of the following program segments: [3]

(1)
```
int a = 12, b = 5, *pa,* pb;
pa = &a;  a = a++ + b;
pb = &b;  b = *pa + 3;
```
(2)
```
int a = 11, b = 13, *pa,*pb;
pb = &b;  a += *pb / 5;
pa = &a;  *pb = a*4;
```

(ii) What is a pointer? How does a pointer variable be declared? If int *x, *y; is a declaration, write the meaning of x+2 and y – 3. [3]

P. T. O.

4. a) (i) Define a function to add two matrices. [2]

(ii) Define a function $f(x,y)=\begin{cases} e^{x+y} & if\ x \le y \\ e^{x*y} & if\ x > y \end{cases}$. [2]

(iii) Discuss bubble sorting. [2]

b) (i) Write a C- program to find $\int_0^2 x^2 e^x dx$ by using rectangle rule. [2]

(ii) Write a C-program to draw x-axis and y-axis and the graph of y = sin(x), by considering center of the screen as the origin. [2]

(iii) Define a function max to find the maximum of two numbers. [2]

OR

b) (i) Define a functions which finds a^n, where a is a real number and n is a natural number. [2]

(ii) Write a C-program to solve y'= –x/y, in the interval [0, 5) by Euler's method (given that y(0) = 5) [2]

(iii) Write a C-program to draw x-axis and y-axis and the graph of $y = e^x$, by considering center of the screen as the origin. [2]

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[28/A-20]

SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester III) Examination

Date: 21-04-2017 **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH25 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) For $G = K_n$, if diam(G) = D and rad(G) = R, then
(a) D < R (b) D > R (c) D = R (d) D = 2R

(3) Let T be a spanning in-tree with root R. Then
(a) $d^+(R) > 0, d^-(R) > 0$ (c) $d^+(R) = 0, d^-(R) = 0$
(b) $d^-(R) = 0, d^+(R) > 0$ (d) $d^+(R) = 0, d^-(R) > 0$

(3) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$
(a) e (b) e^2 (c) 2e (d) ne

(3) For $G = C_5$ with clockwise direction, rank(B) is
(a) 0 (b) 1 (c) 4 (d) 5

(5) The coefficient c_6 in chromatic polynomial of K_6 is
(a) 0 (b) 1 (c) 6! (d) 6

(6) Which of the following graphs is not uniquely colorable?
(a) P_5 (b) K_5 (c) $K_{5,5}$ (d) C_5

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) perfect $\Rightarrow$ maximum (c) maximal $\Rightarrow$ maximum
(b) maximum $\Rightarrow$ perfect (d) maximal $\Rightarrow$ perfect

(8) If $G = P_{11}$, then
(a) $\alpha(G) = \beta(G)$. (b) $\alpha(G) = \beta'(G)$ (c) $\alpha'(G) = \beta'(G)$ (d) none of these

2. Attempt any SEVEN: [14]

(a) Find the radius of $K_{m,n}$ $(m, n \geq 2)$.
(b) Find $|E(G)|$, if G is a complete, symmetric digraph with n vertices,
(c) Prove or disprove: A balanced digraph is regular.
(d) Give an example of a spanning in-tree which is also a spanning out-tree.
(e) Define adjacency matrix of a digraph.
(f) Define Hamiltonian cycle and Hamiltonian graph.
(g) Prove or disprove: The graph P_4 is isomorphic to $K_{1,3}$
(h) Prove or disprove: For any graph G, $\alpha(G) = \beta(G)$.
(i) Prove or disprove: An edge-cover in a graph is a matching.

3. (a) Prove that if G is a connected Euler digraph, then it is balanced. [6]

(b) Define symmetric and complete symmetric digraph and give one example of each. Also, discuss the relation between them. [6]

OR

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \dots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \dots n$. [6]

4. (a) Let G be a connected digraph with n vertices. Prove that rank of $A(G) = n - 1$. [6]

(b) Define spanning out-tree, spanning in-tree and give one example of each. [6]

OR

(b) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is 1, –1 or 0. [6]

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Find the chromatic polynomial for graph $G = K_{1,3}$. [6]

OR

(b) Prove: For a connected graph G, $\chi(G) = 2$ if and only if G has no odd cycle. [6]

6. (a) Prove: A matching M in graph G is maximum if and only if G has no M-augmenting path. [6]

(b) Let G be a graph (no isolated vertex) with n vertices. Prove that $\alpha'(G) + \beta'(G) = n$. [6]

OR

(b) Define $\alpha(G)$ and $\alpha'(G)$ and find it with the corresponding sets, for $G = P_7$. [6]

x-x-x-x-x-x