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Seat No. ______

[35]

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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - II Examination

Friday, 02nd November, 2018

PS02EMTH22, Mathematical Classical Mechanics

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: (1) Figures to the right indicate marks of the respective question.
(2) Assume usual/standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions. [08]

1. A particle moving in space has ______ constraint.
   (a) no (b) a holonomic (c) a rheonomic (d) a scleronomic
2. Lagrangian of a particle of mass m moving on Y-axis is ______.
   (a) $\frac{1}{2}m\dot{y}^2 - mgy$ (b) $\frac{1}{2}m\dot{y}^2 - mgz$ (c) $\frac{1}{2}m\dot{y}^2$ (d) $\frac{1}{2}m\dot{x}^2 + mgx$
3. If $\frac{\partial L}{\partial q_j} = 0$, then ______.
   (a) q_j is cyclic (b) $p_j = 0$ (c) q_j is conserved (d) L is conserved
4. The condition for extremum for the integral $\int f(x, \dot{x}, y)\, dy$ is ______
   (a) $\dfrac{\partial f}{\partial \dot{x}} = \dfrac{d}{dy}\left(\dfrac{\partial f}{\partial x}\right)$ (c) $\dfrac{\partial f}{\partial x} = \dfrac{d}{dy}\left(\dfrac{\partial f}{\partial \dot{x}}\right)$
   (b) $\dfrac{\partial f}{\partial y} = \dfrac{d}{dx}\left(\dfrac{\partial f}{\partial \dot{y}}\right)$ (d) none of these
5. Which of the following is correct?
   (a) $\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$ (b) $\frac{\partial L}{\partial q_j} = \frac{\partial H}{\partial q_j}$ (c) $\frac{\partial L}{\partial t} = \frac{dh}{dt}$ (d) $\frac{\partial L}{\partial q_j} = -\frac{\partial H}{\partial q_j}$
6. If all the generalized coordinates in a system are non-cyclic, then ______.
   (a) $R = L$ (b) $L = -R$ (c) $H = R$ (d) $R = 0$
7. $[u, [v, w]] + [v, [w, u]] =$ ______.
   (a) 0 (b) $[w, [u, v]]$ (c) $[[u, v], w]$ (d) $[u, v]$
8. ______ is not a canonical invariant.
   (a) Generalized velocity (c) Poisson bracket
   (b) Poincare integral (d) Lagrange bracket

Q-2 Attempt *any seven* of the following. [14]

(a) Define a holonomic constraint and give its example.

(b) Define a rigid body and state its degrees of freedom in with more than 3 particles.

(c) Show that generalized momentum conjugate to a cyclic coordinate is conserved.

(d) State law of conservation of angular momentum in Lagrangian formalism.

(e) State Hamilton's principle.

(f) State Hamilton's equations of motion in matrix form.

(g) State transformation equations for a generating function of type $F_3(p, Q, t)$.

(h) In usual notations, show that $[au + bv, w] = a[u, w] + b[v, w]$.

(i) Show that a symplectic matrix is invertible.

(P.T.O)

Q-3 (a) State Lagrange's equations of motion in general form and derive the form in case conservative forces and velocity independent potential. [06]

(b) Obtain Lagrange's equations of motion for a spherical pendulum. [06]

OR

(b) Define virtual displacement and virtual work for a system with N-particles. Hence show that if the system is in equilibrium and constraints are workless, then the total virtual work done by the applied forces vanishes. [06]

Q-4 (a) State and prove the law of conservation of energy in Lagrangian formalism. [06]

(b) Using calculus of variations, show that the shortest distance between two points in a plane is given by a straight line. [06]

OR

(b) Compute energy function and generalized momenta of a system with Lagrangian [06]

$$L = \frac{I_1}{2}(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}\cos\theta)^2 - mgl\cos\theta,$$

where θ, ϕ, ψ are generalized coordinates. Which quantities are conserved? Why?

Q-5 (a) Derive Hamilton's equations of motion from Hamilton's modified principle. [06]

(b) Derive Hamilton's equations of motion for a system with Lagrangian given by [06]

$$L = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{1}{r}.$$

OR

(b) Show that Hamiltonian-like formalism can be set up in which $\dot{q}_i$ and $\dot{p}_i$ are the independent variables with a Hamiltonian $G(\dot{q}, \dot{p}, t)$. Starting from the Lagrangian, construct $G(\dot{q}_i, \dot{p}_i, t)$ and hence derive the corresponding Hamilton's equations of motion. [06]

Q-6 (a) Show that Poisson brackets of two constants of motion is a constant of motion. [06]

(b) Define canonical transformation. Show that the the following transformation is canonical. [06]

$$Q = q\tan p, \ P = \log\sin p.$$

OR

(b) Hamiltonian for a particle moving with constant acceleration a is given by [06]

$$H = \frac{p^2}{2m} - max,$$

where m is the mass of the particle, x is the generalized coordinate, p is momentum conjugate to x. Solve this mechanical system subject to the conditions $x = 0$, $p_0 = mv_0$ at time $t = 0$.

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