[34] SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester II) Examination

Date: 2-11-2018, Friday **Time: 10.00 To 1.00 p.m.**

Subject: MATHEMATICS **Paper No. PS02EMTH21 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) If $K_{1,n} = K_{n+1}$, then
(a) $n = 1$ (b) $n = 2$ (c) $n > 2$ (d) none of these

(2) A symmetric digraph is
(a) Euler (b) connected (c) balanced (d) none of these

(3) For $G = C_n$ with clockwise direction, rank(A) is
(a) 1 (b) $n - 1$ (c) n (d) none of these

(4) If G is a complete symmetric digraph with n vertices, then $|E(G)| =$
(a) $\frac{n(n-1)}{2}$ (b) n (c) $n(n-1)$ (d) n^2

(5) The coefficient c_2 in chromatic polynomial of C_7 is
(a) 7! (b) 7 (c) 1 (d) 0

(6) Which of the following graphs is not Hamiltonian?
(a) K_n (b) $K_{n,n}$ (c) P_n (d) C_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum
(b) perfect $\Rightarrow$ maximal (d) maximal $\Rightarrow$ perfect

(8) If $G = K_{3,n}$, then $\beta(G) =$ ____.
(a) 3 (b) n (c) $\min\{3, n\}$ (d) $\max\{3, n\}$

2. Attempt any SEVEN: [14]

(a) Find the radius of $K_{m,n}$ $(m, n \geq 2)$.

(b) Prove or disprove: A balanced digraph is connected.

(c) Define adjacency matrix in a digraph and give one example of it.

(d) Prove or disprove: Every connected digraph has a spanning out-tree.

(e) Prove: If G is a bipartite graph, then $\chi(G) = 2$.

(f) What is Four color problem?

(g) Prove or disprove: The graph C_4 is isomorphic to $K_{2,2}$.

(h) Prove: If $S \subset V(G)$ is a vertex cover, then $V(G) - S$ is an independent set, in G.

(i) Define perfect matching and give one perfect matching in P_8.

(P.T.O.)

3. (a) Prove that if G is connected Euler digraph, then it is balanced.

(b) Define spanning in-tree, spanning out-tree and give one example of each in the same digraph. [6]

OR

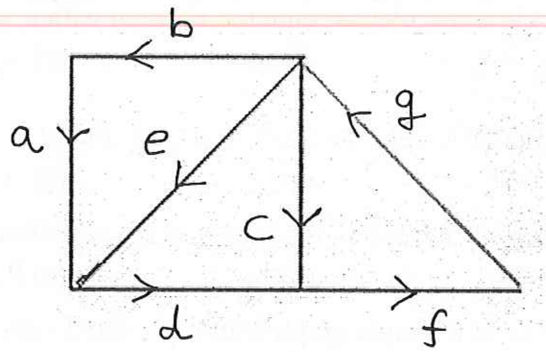
(b) Define the following digraphs with examples: [6]
(i) Asymmetric & complete asymmetric (ii) Symmetric & complete symmetric.

4. (a) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

(b) Prove that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Define a fundamental circuit matrix in a digraph and find it w. r. t. spanning tree T = {a, c, d, f} in digraph below: [6]



5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Let G be a k-chromatic graph with n vertices. Prove that $n \leq k\,\alpha(G)$. [6]

OR

(b) Define chromatic number $\chi(G)$ of a graph G. Give an example of a non-complete graph G with $\chi(G) = \Delta(G) + 1$. [6]

6. (a) Prove: If G is a bipartite graph, then $\alpha'(G) = \beta(G)$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G)$, $\beta(G)$ and find it with the corresponding sets for $G = K_{n,m}$ $(n \neq m)$. [6]

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