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Seat No. ______

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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - II Examination

Friday, 02nd November, 2018

PS02EMTH04, Mathematical Classical Mechanics

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: (1) Figures to the right indicate marks of the respective question.
(2) Assume usual/standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions. [08]

1. If work done due to force $\bar{F}$ does not depend on the path, then $\bar{F}$ is ______.
   (a) λV (b) $-\nabla \times V$ (c) conservative (d) zero
2. For a system of particles center of mass ______.
   (a) is unique (c) does not exists
   (b) is stationary (d) is not unique
3. ______ gives the shortest distance between two particles in space.
   (a) Straight line (b) Catenary (c) Cycloid (d) Great circle
4. If Lagrangian does not depend on q explicitly, then ______ is conserved.
   (a) L (b) $\dot{q}$ (c) p (d) H
5. Which of the following is incorrect?
   (a) $\frac{\partial L}{\partial t} = -\frac{\partial H}{\partial t}$ (b) $\frac{\partial L}{\partial q_j} = -\frac{\partial H}{\partial q_j}$ (c) $\frac{\partial L}{\partial t} = -\frac{dh}{dt}$ (d) $\frac{\partial L}{\partial q_j} = \frac{\partial H}{\partial q_j}$
6. If q_j is cyclic in L, then it is ______ in H.
   (a) non-cyclic (b) ignorable (c) zero (d) constant
7. $[q_2 + q_3, p_2] =$ ______; notation being usual.
   (a) -1 (b) 1 (c) 0 (d) 2
8. Lagrange bracket is ______.
   (a) constant (c) always non-vanishing
   (b) symmetric (d) a canonical invariant

Q-2 Attempt *any seven* of the following. [14]

(a) Define a non-holonomic constraint and give its example.

(b) Define center of mass for a system of particles.

(c) Define cyclic coordinate

(d) State law of conservation of energy in Lagrangian formalism.

(e) In usual notations, show that $\frac{\partial H}{\partial t} = \frac{dH}{dt}$.

(f) State the principle of least action.

(g) State the condition for a canonical transformation generated by a generating function of type $F_2(q, P, t)$.

(h) Show that the transformation $Q_i = p_i$, $P_i = -q_i$ is canonical.

(i) Let H be Hamiltonian of a system. Show that if $u(q, p, t)$ is a constant of motion, then $\frac{\partial u}{\partial t} = [H, u]$.

(P.T.O)

Q-3 (a) State Lagrange's equations of motion in general form. Hence derive Lagrange's equations of motion for a system when the forces are conservative and potential is independent of velocities. [06]

(b) Obtain Lagrange's equations of motion for a simple pendulum. [06]

OR

(b) For a system with n-degrees of freedom show that $L' = L + \frac{dF}{dt}$ satisfies Lagrange's equations of motion, where L is Lagrangian of the system and $F(q_1, q_2, \cdots, q_n, t)$ is an arbitrary differentiable function of its arguments. [06]

Q-4 (a) State and prove the law of conservation of linear momentum in Lagrangian formalism. [06]

(b) Derive Euler's equation as the condition for extremum of the integral $\int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$. [06]

OR

(b) Lagrangian of a system is given by [06]

$$L = \frac{m}{2}\left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2\right) - \frac{k}{2}\left(ax^2 + 2bxy + cy^2\right).$$

Evaluate the energy function and conjugate momenta. Which of them conserved? Justify.

Q-5 (a) State Hamilton's modified principle and derive Hamilton's equations of motion from it. [06]

(b) Derive Routhian equations of motion for a system with Lagrangian given by [06]

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r}.$$

OR

(b) State Hamilton's equations of motion in matrix form and hence verify them for a system with degrees of freedom $n = 3$. [06]

Q-6 (a) Define a symplectic matrix. Show that the set of symplectic matrices forms a group under matrix multiplication. [06]

(b) Define canonical transformation. Show that the transformation [06]

$$Q_1 = q_1, \quad P_1 = p_1 - 2p_2, \quad Q_2 = p_2, \quad P_2 = -2q_1 - q_2.$$

is canonical.

OR

(b) Using Poisson brackets, show that no two components of angular momentum $\bar{L}$ can simultaneously be chosen as canonical variables. [06]

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