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[32] SEAT No.________

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SARDAR PATEL UNIVERSITY
M. Sc. (Semester II) Examination

Date: 2-11-2018, Friday **Time: 10.00** AM **To 01.00** PM
Subject: MATHEMATICS **Paper No. PS02EMTH02 – (Graph Theory – I)**
Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The radius of K_n $(n > 3)$ is
(a) 1 (b) 2 (c) n (d) 0

(2) Let T be a spanning in-tree with root R. Then
(a) $d^-(R) = 0$ (b) $d^-(R) > 0$ (c) $d^+(R) > 0$ (d) none of these

(3) For $G = C_n$ with clockwise direction, rank(B) is
(a) n (b) $n - 1$ (c) 1 (d) none of these

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$
(a) ne (b) 2e (c) e^2 (d) e

(5) The coefficient c_4 in chromatic polynomial of K_4 is
(a) 0 (b) 1 (c) 4 (d) 4!

(6) Which of the following graphs is Hamiltonian?
(a) $K_{n,2n}$ (b) P_{2n} (c) C_n (d) P_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum
(b) maximum $\Rightarrow$ maximal (d) maximum $\Rightarrow$ perfect

(8) For which of the following graphs, $\alpha'(G) = \beta(G)$?
(a) P_9 (b) K_{10} (c) K_{11} (d) C_{13}

2. Attempt any SEVEN: [14]

(a) Find the diameter of $K_{3,4}$.
(b) Prove or disprove: An Euler digraph is connected.
(c) Define circuit matrix in a digraph.
(d) Prove or disprove: Every connected digraph has a spanning in-tree.
(e) Prove: If $G = C_n$ and n is odd, then $\chi(G) = 3$.
(f) What is Four color problem?
(g) Why P_4 is not isomorphic to $K_{1,3}$?
(h) Prove: If $S \subset V(G)$ is an independent set, then $V(G) - S$ is is a vertex cover in G.
(i) Define an edge cover of a graph and give one example of it.

[P.T.O.]

3. (a) Define the following with examples: [6]
(i) In-degree (ii) Out-degree (iii) Balanced digraph (iv) Regular digraph

(b) Prove that if G is connected Euler digraph, then it is balanced. [6]

OR

(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is $1, -1$ or 0. [6]

(b) Prove that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \ldots n$. [6]

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Let G be a k-chromatic graph with n vertices. Prove that $n \leq k\,\alpha(G)$. [6]

OR

(b) Find the coefficients c_2 and c_3 of Chromatic polynomial of graph $K_{2,2}$. [6]

6. (a) Let G be a graph (no isolated vertex) with n vertices. Prove that $\alpha'(G) + \beta'(G) = n$. [6]

(b) State Hall's theorem & show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G)$, $\beta(G)$ and find it with the corresponding sets for $G = P_7$. [6]

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