SEAT No.________

[55]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-II), PS02CMTH25, Methods of Partial Differential Equations;
Tuesday, 30^{th} October, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of $(D+D')(D'-D)^2z=0$ is
   (A) 2 (B) 3 (C) 4 (D) 1
2. The equation $r-t=0$ can be written in the form $F(D,D')z=0$, where $F(D,D')$ equals
   (A) $D^2-D'^2$ (B) D'^2-D^2 (C) D'^2-DD' (D) D^2-DD'
3. The complete integral of $z=px+qy+pq$ is
   (A) $z=ax+by+ab$ (B) $z=ap+bq+pq$
   (C) $z=ax^2+by^2+ab$ (D) none of these
4. Let $u=\log x$ and $v=\log y$ in $z=z(x,y)$. Then $y^2\frac{\partial^2 z}{\partial y^2}$ becomes
   (A) $\frac{\partial^2 z}{\partial v^2}-\frac{\partial z}{\partial v}$ (B) $\frac{1}{y}\left(\frac{\partial^2 z}{\partial v^2}-\frac{\partial z}{\partial v}\right)$
   (C) $\frac{1}{y^2}\left(\frac{\partial^2 z}{\partial v^2}-\frac{\partial z}{\partial v}\right)$ (D) none of these
5. The equation $x^2r+2xys+y^2t=0$ is elliptic, if
   (A) $|x|=2$ (B) $|x|<2$ (C) $|x|>2$ (D) none of these
6. In Monge's method, the $\lambda-$ quadratic equation of $3s+rt-s^2=2$ is
   (A) $2\lambda^2+3\lambda+1=0$ (B) $2\lambda^2-3\lambda+1=0$
   (C) $\lambda^2+3\lambda+1=0$ (D) none of these
7. The solution of $x^2y''+xy'+(x^2-m^2)y=0$ is
   (A) $J_m(x)$ (B) $J_m(mx)$ (C) $J(m^2x)$ (D) none of these
8. The two dimensional wave equation is
   (A) $u_{xx}+u_{yy}=0$ (B) $u_{xx}+u_{yy}=\frac{1}{c^2}u_t$
   (C) $u_{xx}+u_{yy}=\frac{1}{c^2}u_{tt}$ (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define general solution of partial differential equation.
(b) Find a pde by eliminating f and g from $z=f(x-y)+g(x+y)$.
(c) Give an example of pde whose general solution is $\phi_1(x+y)+\phi_2(x-y)$, where ϕ_1 and ϕ_2 are arbitrary functions.
(d) Find D^2z, if x and y in $z=z(x,y)$ replaced by $u=\log x$ and $v=\log y$.
(e) Show that $p=F(x,y)$ and $q=G(x,y)$ are compatible if $\frac{\partial F}{\partial y}=\frac{\partial G}{\partial x}$.
(f) Give an example of pde which is hyperbolic in region $\{(x,y)\in\mathbb{R}^2:|x|<2\}$.
(g) Find $u=u(x,y)$ and $v=v(x,y)$ to covert $r-t=0$ in the canonical form.
(h) Write heat equation in spherical coordinates.
(i) State Neumann interior BVP for a circle.

Q.3

(a) If $(\beta D' + \gamma)^2$ $(\beta \neq 0)$ is a factor of $F(D, D')$, then prove that $e^{-\frac{\gamma}{\beta}y}[\phi_1(\beta x) + y\phi_2(\beta x)]$ is a solution of $F(D, D')z = 0$, where ϕ_1 and ϕ_2 are arbitrary functions. [6]

(b) Find the general solution of $(D^2 - D'^2 - 3)z = e^{2x+y}$. [6]

OR

(b) Find particular integral of $(D^2 + 2DD' + D'^2)z = x \sin y$.

Q.4

(a) Find the general solution of $(x^2D^2 - y^2D'^2 - yD' + xD)z = xy^2$. [6]

(b) Find the complete integral of $(p^2 + q^2)y - qz = 0$ using Charpit's method. [6]

OR

(b) Find the complete integral of $z^2 = pqxy$ using Jacobi's method.

Q.5

(a) Convert $r + x^2t = 0$ into the canonical form. [6]

(b) Solve $r - 4t = 0$ using Monge's method. [6]

OR

(b) Solve $3r + 4s + t + rt - s^2 - 1 = 0$ using Monge's method.

Q.6

(a) Solve $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{1}{k}\frac{\partial \varphi}{\partial t}$ by the method of separation of variables and show that the solution can be put in the form of $\varphi(x, y, t) = e^{i(nx+my)-k(n^2+m^2)t}$, where n and m are constants. [6]

(b) Derive Laplace equation in cylindrical coordinates. [6]

OR

(b) Find $u = u(x, y)$ such that $\nabla^2 u = 0$ in $\{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$ with

$$\begin{aligned} u(x,0) &= f(x), \quad 0 \leq x \leq a \\ u(a,y) &= 0, \quad 0 \leq y \leq b \\ u(x,b) &= 0, \quad 0 \leq x \leq a \\ u(0,y) &= 0, \quad 0 \leq y \leq b. \end{aligned}$$

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