[98] SEAT No.________ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-II) Examination
October-2018
Thursday, October25, 2018
Time: 10:00 AM to 01:00 PM

Subject: Mathematics **Course No.PS02CMTH23**

Differential Geometry

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) A Cartesian representation of the curve $\bar{\gamma}(t) = (\sin^2 t, \cos^2 t)$
(a) $x + y = 1$ (b)$x - y = 1$ (c) $x^{2/3} + y^{2/3} = 1$ (d) none of these

(2) Which of the following represents a helix?
(a) $(b\sin t,\ b\,t,\ c\cos t)$ (b) (t, t^2, t^3)
(c) $(b\,t, b\sin t, b\cos t)$ (d) none of these

(3) Which one of the following is a plane curve?
(a) circle (b) helix (c) parabola (d) none of these

(4) Which one of the following can be covered by a single patch?
(a) sphere (b) ellipsoid (c) hyperboloid (d) none of these

(5) The equation of the tangent space to $x^2 + y^2 + z^2 = 4$ at the point (0,2,0)is
(a) $y = 2$ (b) $z = 0$ (c) $y = 0$ (d) $z = 2$

(6) Which one of the following is not a smooth surface?
(a) $x^2 + y^2 + z^2 = 1$ (b) $x^2 + y^2 - z^2 = 1$
(c) $x^2 + z^2 = y$ (d) none of these

(7) The point (0,0,0)on the surface $z = x^2 + y^2$is
(a) parabolic (b) elliptic (c) hyperbolic (d) flat

(8) Which one of the following is correct?
(a) $\sigma_{uu}\sigma_u = E_u$ (b) $\sigma_{uu}\sigma_u = \frac{\pi}{2}E_u$ (c) $\sigma_{uu}\sigma_u = \frac{1}{2}E_u$ (d) $\sigma_{uu}\sigma_u = 0$

Q-2 Answer any **Seven.** (14)

(1) Compute arc-length of the curve $\bar{\gamma}(t) = (3\cos t, 3\sin t)$ between the points $\bar{\gamma}(0)$ to $\bar{\gamma}(\pi)$.
(2) Define a parametrized curve in the space R^n.
(3) Find the tangent to the curve $\bar{\gamma}(t) = (t, t^2, t^3)$ at the point (1,1,1).
(4) State isoperimetric inequality for space curves.
(5) What is the surface represented by the equation $y^2 - x^2 = 1$in R^3?
(6) Compute the surface area of $\{(x, y, z) \in R^3 : x^2 + z^2 = 1, y \in (-1,1)\}$.
(7) Define second order magnitudes on a surface.
(8) State the condition for a unit speed curve on a surface to be geodesic.
(9) State Bonnet's theorem.

(PTO)

Q-3

(a) Let $\bar{\gamma}(t)$ be a regular curve in R^3 with nowhere vanishing curvature. Show that it is spherical if and only if $\frac{\tau}{\kappa} = \frac{d}{dt}\left(\frac{\dot{\kappa}}{\kappa^2 \tau}\right)$. (06)

(b) Let p and q be positive reals. Show that $\int_0^{2\pi} \sqrt{p^2 \sin^2 t + q^2 \cos^2 t}\ dt \geq 2\pi\sqrt{pq}$. Also show that equality holds if and only if $p = q$. (06)

OR

(b) For a regular curve $\bar{\gamma}$, in usual notations show that $\kappa = \frac{\|\ddot{\bar{\gamma}} x \dot{\bar{\gamma}}\|}{\|\dot{\bar{\gamma}}\|^3}$.

Q-4

(a) Show that $\Sigma = \{(x, y, z) \in R^3 : z = y^2 - x^2\}$ is surface. (06)

(b) If a smooth map $f: S_1 \to S_2$ is a local isometry, then show that $\langle .,. \rangle_p = f^*\langle .,. \rangle_p$ on $T_p S_1$ for all $p \in S_1$. (06)

OR

(b) Define derivative of a function between two surfaces. State and prove chain rule for such derivative.

Q-5

(a) Compute the second fundamental form on the surface $\{(x, y, z) \in R^3 : z = x^2 + y^2\}$. (06)

(b) Let σ be a surface patch of an oriented surface with the unit normal $\bar{N}$, show that $\bar{N}_u \sigma_u = -L$, $\bar{N}_u \sigma_v = -M = \bar{N}_v \sigma_u$, $\bar{N}_v \sigma_v = -N$; notations being usual. (06)

OR

(b) Compute Gaussian curvature and mean curvature of $\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$, where $\left(\frac{df}{du}\right)^2 + \left(\frac{dg}{du}\right)^2 = 1$

Q-6

(a) Determine geodesics on a plane. (06)

(b) Let σ be a surface patch of an oriented surface S. Show that (06)
$L_v - M_u = L\Gamma_{12}^1 + M(\Gamma_{12}^2 - \Gamma_{11}^1) - N\Gamma_{11}^2$ and
$M_v - N_u = L\Gamma_{22}^1 + M(\Gamma_{22}^2 - \Gamma_{12}^1) - N\Gamma_{12}^2$.

OR

(b) Compute Christoffel symbols of second kind for the circular cylinder $\sigma(u, v) = (\cos u, \sin u, v)$.

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