(33) SEAT No.________

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Sardar Patel University

Mathematics

M.Sc. Semester II

Saturday, 20 October 2018

10.00 a.m. to 01.00 p.m.

PS02CMTH21 - Real Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) $m^*([-1,1]\cap(\mathbb{R}-\mathbb{Q})) = \ldots\ldots$

(a) 0 (b) 1 (c) 2 (d) ∞

(2) Let E be a nonmeasurable subset of $\mathbb{R}$. Which of the following functions is measurable?

(a) χ_E (b) $1-\chi_E$ (c) $(\chi_E)^2$ (d) $\chi_E + \chi_{\mathbb{R}-E}$

(3) Which of the following functions is not Riemann integrable over $[0,1]$?

(a) $x\sin\frac{1}{x}$ (b) $\sin x$ (c) $x\cos\frac{1}{x}$ (d) $\frac{1}{x}$

(4) Let $f = \chi_E + \chi_F$, where E and F are disjoint bounded measurable sets. Then the value of $\int_{E\cup F} f$ is

(a) $mE + mF$ (b) $m(E)m(F)$ (c) $mE - mF$ (d) $m(E\cap F)$

(5) Let $f(x) = \sin x$ for $x \in [0,\pi]$. Then $\int_0^\pi f^-$ is ____

(a) 1 (b) 0 (c) π (d) $\frac{\pi}{2}$

(6) Let f be integrable over a measurable set E. Which of the following is not true?

(a) $\int_E f \le \int_E |f|$ (b) $-\int_E f \le \int_E |f|$ (c) $|\int_E f| \le \int_E |f|$ (d) $|\int_E f| \ge \int_E |f|$

(7) Let $f(x) = x^2$ for all $x \in [0,1]$. Then the total variation of f over $[0,1]$ is

(a) 0 (b) 1 (c) 2 (d) 4

(8) Let $f : [a,b] \to \mathbb{R}$. Which of the following is true?

(a) If f is bounded, then it is of bounded variation.

(b) If f is continuous, then it is of bounded variation.

(c) If f is differentiable, then it absolutely continuous.

(d) If f' is bounded, then f is absolutely continuous.

Q.2 Attempt any ***Seven***. [14]

(a) Show intersection of two σ- algebras on a set X is a σ- algebra.

(b) Find the measure of the set $\{\pi + x : x \in \mathbb{R}-\mathbb{Q}\}$.

(c) If $|f|$ is measurable over $\mathbb{R}$, then show that f need not be measurable.

(d) If s and t are simple functions, then show that $s+t$ is a simple function.

(e) If a nonnegative measurable function f is integrable over a measurable set E, then show that f is finite a.e. in E.

(f) If f is integrable over E and $\alpha \in \mathbb{R}$, then show that $\int_E \alpha f = \alpha \int_E f$.

(P.T.O)

(g) Let $\{f_n\}$ be a sequence of nonnegative measurable functions defined on a measurable set E and $f_n \to f$ on E. If $f_n \le f$ for each n, then show that $\int_E f = \lim_n \int_E f_n$.

(h) If f is of bounded variation on $[a, b]$ and $\alpha \in \mathbb{R}$, then show that $T_a^b(\alpha f) = |\alpha| T_a^b(f)$.

(i) Let $f : [0, 1] \to \mathbb{R}$ be $f(x) = x^2$. Show that f is absolutely continuous.

Q.3

(a) Let $A \subset \mathbb{R}$ and $\epsilon > 0$. Show that there is an open set O such that $A \subset O$ and $m^*O \le m^*A + \epsilon$. Also, show that there is a G_δ set G such that $A \subset G$ and $m^*A = m^*G$. [6]

(b) Let f be an extended real valued function on a measurable set E. Show that f is measurable if and only if $\{x \in E : f(x) < r\}$ is measurable for every $r \in \mathbb{Q}$. [6]

OR

(b) If $\{E_n\}$ is a decreasing sequence of measurable subsets of $\mathbb{R}$ and if $mE_1 < \infty$, then show that $m(\bigcap_{n=1}^{\infty} E_n) = \lim_{n \to \infty} mE_n$. [6]

Q.4

(c) Let E be a measurable set of finite measure, and $\{f_n\}$ a sequence of measurable functions defined on E. Let f be a real valued function such that $f_n(x) \to f(x)$ for each x in E. Show that given $\epsilon > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ with $mA < \delta$ and an integer N such that $|f_n(x) - f(x)| < \epsilon$ for all $x \notin A$ and all $n > N$. [6]

(d) If φ and ψ are measurable simple functions vanishing outside a set of finite measure and if $a, b \in \mathbb{R}$, then show that $\int (a\varphi + b\psi) = a \int \varphi + b \int \psi$. [6]

OR

(d) Let f be a real valued measurable function on E and $mE < \infty$. Show that for each $\epsilon > 0$, there is a continuous function g on $\mathbb{R}$ and a closed set F contained in E such that $f = g$ on F and $m(E - F) < \epsilon$. [6]

Q.5

(e) If f and g are integrable over a measurable set E, then show that $f + g$ is integrable over E and $\int_E (f + g) = \int_E f + \int_E g$. [6]

(f) Let E be a measurable set, and let f_n and f be measurable functions on E. Show that $f_n \to f$ in measure on E if and only if $m\{x \in E : |f_n(x) - f(x)| \ge \sigma\} \to 0$ as $n \to \infty$ for all $\sigma > 0$. [6]

OR

(f) Let f be integrable over E. Show that for given $\epsilon > 0$, there is $\delta > 0$ such that $|\int_F f| < \epsilon$ whenever F is a measurable subset of E with $mF < \delta$. [6]

Q.6

(g) If f is absolutely continuous on $[a, b]$ and $f' = 0$ a.e., then show that f is constant. [6]

(h) Let f be an integrable function on $[a, b]$, and let $F(x) = F(a) + \int_a^x f$ for all $x \in [a, b]$. Show that $F'(x) = f(x)$ for almost all x in $[a, b]$. [6]

OR

(h) If f is of bounded variation on $[a, b]$ and $a < c < b$, then show that f is of bounded variation on both $[a, c]$ and $[c, b]$. Also, show that $T_a^b(f) = T_a^c(f) + T_c^b(f)$. [6]

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