

[54]

**Sardar Patel University**

M.Sc. (Sem-II), PS02CMTH05, Methods of Partial Differential Equations;  
 Tuesday, 30<sup>th</sup> October, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

**Note:** (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following.

[8]

- The order of  $(D - 1)(D'^2 - D)z = 0$  is  
 (A) 2 (B) 3 (C) 4 (D) 1
- The equation  $r - 2s = 0$  can be written in the form  $F(D, D')z = 0$ , where  $F(D, D')$  equals  
 (A)  $D^2 - 2DD'$  (B)  $D^2 - D'^2$  (C)  $D^2 - 2D'^2$  (D)  $D'^2 - 2D^2$
- The equation  $x^2r - 2s + t = 0$  is classified as elliptic on  
 (A)  $|x| > 1$  (B)  $|x| < 1$  (C)  $|x| = 1$  (D) none of these
- In Monge's method, the  $\lambda$ -quadratic equation of  $3r + 4s + t + rt - s^2 = 1$  is  
 (A)  $4\lambda^2 + 4\lambda + 1 = 0$  (B)  $\lambda^2 - 4\lambda + 1 = 0$   
 (C)  $4\lambda^2 - 4\lambda + 1 = 0$  (D) none of these
- The solution of  $x^2y'' + xy' + (a^2x^2 - m^2)y = 0$  is  
 (A)  $J_m(ax)$  (B)  $J_a(mx)$  (C)  $J_m(x)$  (D)  $J_a(x)$
- Which one from the following is wave equation?  
 (A)  $u_{xx} + u_{yy} + u_{zz} = 0$  (B)  $u_{xx} + u_{yy} = \frac{1}{c^2}u_{tt}$   
 (C)  $u_{xx} = \frac{1}{k}u_t$  (D) none of these
- If  $u$  and  $v$  are any two solutions of Neumann BVP, then  
 (A)  $u - v = \alpha$  ( $\alpha \in \mathbb{R}$ ) (B)  $u = \alpha v$  ( $1 \neq \alpha \in \mathbb{R}$ )  
 (C)  $u = v$  (D) none of these
- If  $u$  and  $v$  are any two solutions of Dirichlet BVP, then  
 (A)  $u = \alpha v$  ( $1 \neq \alpha \in \mathbb{R}$ ) (B)  $u - v = \alpha$  ( $0 \neq \alpha \in \mathbb{R}$ )  
 (C)  $u = v$  (D) none of these

Q.2 Attempt any *seven*:

[14]

- Define complementary function of pde.
- Find a pde by eliminating  $f$  and  $g$  from  $z = f(x - iy) + g(x + iy)$ .
- Solve:  $(D^2 - D'^2)z = 0$ .
- Find  $D'^2z$ , if  $x$  and  $y$  in  $z = z(x, y)$  replaced by  $u = \log x$  and  $v = \log y$ .
- Classify the region in which the equation  $4y^2r + x^2t = 0$  is parabolic.
- Give an example of pde which is hyperbolic in region  $\{(x, y) \in \mathbb{R}^2 : |x| < 1\}$ .
- Find  $u = u(x, y)$  and  $v = v(x, y)$  to convert  $r + 2s + t = 0$  in the canonical form.
- State minimum principle.
- State Green's theorem.

Q.3

- (a) If  $(\beta D' + \gamma)^2$  ( $\beta \neq 0$ ) is a factor of  $F(D, D')$ , then prove that  $e^{-\frac{\gamma}{\beta}y}[\phi_1(\beta x) + y\phi_2(\beta x)]$  [6]  
is a solution of  $F(D, D')z = 0$ , where  $\phi_1$  and  $\phi_2$  are arbitrary functions. [6]
- (b) Find the general solution of  $(D^2 - D')z = x - y$ . [6]

OR

- (b) Find a particular integral of  $(2D^2 - 3D + D')z = \sin(2x - y)$ .

Q.4

- (a) Convert  $r - t = 0$  into the canonical form. [6]
- (b) Solve  $r + 3s - 10t = xy$  using Monge's method. [6]

OR

- (b) Solve  $3s + rt - s^2 - 2 = 0$  using Monge's method.

Q.5

- (a) Find the general solution of  $(x^2D^2 - y^2D'^2 - yD' + xD)z = 0$ . [6]
- (b) Solve  $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{1}{k} \frac{\partial \varphi}{\partial t}$  by the method of separation of variables and show that the [6]  
solution can be put in the form of  $\varphi(x, y, t) = e^{i(nx+my) - k(n^2+m^2)t}$ , where  $n$  and  $m$  are  
constants.

OR

- (b) Derive Laplace equation in cylindrical coordinates.

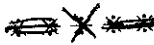
Q.6

- (a) State and prove maximum principle. [6]
- (b) Find  $u = u(x, y)$  such that  $\nabla^2 u = 0$  in  $\{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$  with [6]

$$\begin{aligned}u(x, 0) &= f(x), & 0 \leq x \leq a \\u(a, y) &= 0, & 0 \leq y \leq b \\u(x, b) &= 0, & 0 \leq x \leq a \\u(0, y) &= 0, & 0 \leq y \leq b.\end{aligned}$$

OR

- (b) Discuss the Neumann interior BVP for a circle.



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