SEAT No.________

[54]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-II), PS02CMTH05, Methods of Partial Differential Equations;
Tuesday, 30^{th} October, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of $(D-1)(D'^2-D)z=0$ is
   (A) 2 (B) 3 (C) 4 (D) 1
2. The equation $r-2s=0$ can be written in the form $F(D,D')z=0$, where $F(D,D')$ equals
   (A) D^2-2DD' (B) $D^2-D'^2$ (C) $D^2-2D'^2$ (D) D'^2-2D^2
3. The equation $x^2r-2s+t=0$ is classified as elliptic on
   (A) $|x|>1$ (B) $|x|<1$ (C) $|x|=1$ (D) none of these
4. In Monge's method, the $\lambda-$ quadratic equation of $3r+4s+t+rt-s^2=1$ is
   (A) $4\lambda^2+4\lambda+1=0$ (B) $\lambda^2-4\lambda+1=0$
   (C) $4\lambda^2-4\lambda+1=0$ (D) none of these
5. The solution of $x^2y''+xy'+(a^2x^2-m^2)y=0$ is
   (A) $J_m(ax)$ (B) $J_a(mx)$ (C) $J_m(x)$ (D) $J_a(x)$
6. Which one from the following is wave equation ?
   (A) $u_{xx}+u_{yy}+u_{zz}=0$ (B) $u_{xx}+u_{yy}=\frac{1}{c^2}u_{tt}$
   (C) $u_{xx}=\frac{1}{k}u_t$ (D) none of these
7. If u and v are any two solutions of Neumann BVP, then
   (A) $u-v=\alpha$ $(\alpha\in\mathbb{R})$ (B) $u=\alpha v$ $(1\neq\alpha\in\mathbb{R})$
   (C) $u=v$ (D) none of these
8. If u and v are any two solutions of Dirichlet BVP, then
   (A) $u=\alpha v$ $(1\neq\alpha\in\mathbb{R})$ (B) $u-v=\alpha$ $(0\neq\alpha\in\mathbb{R})$
   (C) $u=v$ (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define complementary function of pde.
(b) Find a pde by eliminating f and g from $z=f(x-iy)+g(x+iy)$.
(c) Solve: $(D^2-D'^2)z=0$.
(d) Find D'^2z, if x and y in $z=z(x,y)$ replaced by $u=\log x$ and $v=\log y$.
(e) Classify the region in which the equation $4y^2r+x^2t=0$ is parabolic.
(f) Give an example of pde which is hyperbolic in region $\{(x,y)\in\mathbb{R}^2:|x|<1\}$.
(g) Find $u=u(x,y)$ and $v=v(x,y)$ to covert $r+2s+t=0$ in the canonical form.
(h) State minimum principle.
(i) State Green's theorem.

[P.T.O.]

Q.3

(a) If $(\beta D' + \gamma)^2$ $(\beta \neq 0)$ is a factor of $F(D, D')$, then prove that $e^{-\frac{\gamma}{\beta}y}[\phi_1(\beta x) + y\phi_2(\beta x)]$ is a solution of $F(D, D')z = 0$, where ϕ_1 and ϕ_2 are arbitrary functions. [6]

(b) Find the general solution of $(D^2 - D'^2)z = x - y$. [6]

OR

(b) Find a particular integral of $(2D^2 - 3D + D')z = \sin(2x - y)$.

Q.4

(a) Convert $r - t = 0$ into the canonical form. [6]

(b) Solve $r + 3s - 10t = xy$ using Monge's method. [6]

OR

(b) Solve $3s + rt - s^2 - 2 = 0$ using Monge's method.

Q.5

(a) Find the general solution of $(x^2D^2 - y^2D'^2 - yD' + xD)z = 0$. [6]

(b) Solve $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{k}\frac{\partial\varphi}{\partial t}$ by the method of separation of variables and show that the solution can be put in the form of $\varphi(x, y, t) = e^{i(nx+my)-k(n^2+m^2)t}$, where n and m are constants. [6]

OR

(b) Derive Laplace equation in cylindrical coordinates.

Q.6

(a) State and prove maximum principle. [6]

(b) Find $u = u(x, y)$ such that $\nabla^2 u = 0$ in $\{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$ with [6]

$$\begin{aligned} u(x,0) &= f(x), \; 0 \leq x \leq a \\ u(a,y) &= 0, \quad 0 \leq y \leq b \\ u(x,b) &= 0, \quad 0 \leq x \leq a \\ u(0,y) &= 0, \quad 0 \leq y \leq b. \end{aligned}$$

OR

(b) Discuss the Neumann interior BVP for a circle.

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(2)