[81/A10] **SEAT No.________**

No of printed pages: 2

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M.Sc. (Sem-II), PS02CMTH02, Algebra-I;

Tuesday, 23^{rd} October, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The number of elements in an integral domain can not be
   (A) 10 (B) 2 (C) 25 (D) 8
2. $\mathbb{Z}_2[x]/ < x^2 + x + 1 >$ is
   (A) a field containing 4 elements (B) an infinite field
   (C) a field containing 8 elements (D) NOT a field
3. The polynomial $x^2 - 7$ is reducible over
   (A) $\mathbb{Z}$ (B) $\mathbb{Q}$ (C) $\mathbb{C}$ (D) none of these
4. Which is not unit in $J[i]$?
   (A) i (B) $-i$ (C) 1 (D) -2
5. The content of a polynomial $8x^2 - 4x + 2$ is
   (A) 2 (B) 4 (C) 8 (D) 1
6. $[\mathbb{R} : \mathbb{Q}] =$
   (A) 1 (B) 2 (C) 3 (D) ∞
7. Which one from the following is not a radical extension of $\mathbb{Q}$?
   (A) $\mathbb{Q}(e)$ (B) $\mathbb{Q}(\sqrt{2})$ (C) $\mathbb{Q}(\sqrt{5})$ (D) $\mathbb{Q}(\sqrt{2}, \sqrt{3})$
8. The group S_n is not solvable for $n =$
   (A) 3 (B) 4 (C) 5 (D) 2

Q.2 Attempt any *seven*: [14]

(a) Define Euclidean ring.
(b) Show that the unit element is associate to any element in the Euclidean ring.
(c) State division algorithm in $F[x]$, where F is a field.
(d) Let $f(x) \in F[x]$ and $a \in F$. If $f(x + a)$ is irreducible over F then show that $f(x)$ is irreducible over F.
(e) Show that units in $F[x]$ are units of F where F is a field.
(f) Find $[\mathbb{Q}(\sqrt{5}) : \mathbb{Q}]$.
(g) Show that every element in $\mathbb{R}$ is algebraic over $\mathbb{R}$.
(h) Define radical extension.
(i) Prove or disprove: every abelian group is solvable.

Q.3

(a) State and prove Unique Factorization Theorem. [6]
(b) Show that every Euclidean ring is a principal ideal ring. [6]

OR

(b) In a Euclidean ring R, show that a non-zero element u in R is a unit if and only if $d(u) = d(1)$.

(P.T.O.)

Q.4

(a) State and prove Eisenstein criterion. [6]

(b) Show that the ideal $< p(x) >$ in $F[x]$ is maximal iff $p(x)$ is irreducible over F. [6]

OR

(b) Construct a field of order 9.

Q.5

(a) If L is a finite extension of K and if K is a finite extension of F then show that L is a finite extension of F. [6]

(b) If L is algebraic extension of K and if K is algebraic extension of F, then show that L is algebraic extension of F. [6]

OR

(b) Find the degree of splitting field of $x^3 - 2$ over $\mathbb{Q}$.

Q.6

(a) Let K be a normal extension of F, H be a subgroup of $G(K, F)$ and K_H be a fixed field of H. Then show that $[K : K_H] = o(H)$. [6]

(b) Show that the group S_n, $n \geq 5$ is not solvable. [6]

OR

(b) State and prove Able's theorem.

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