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# SARDAR PATEL UNIVERSITY

**M.Sc. (Semester-II) Examination**

**October-2016**

**Thursday 27/10/2016**

**Time: 10:00 AM to 1:00 PM**

**Subject: Mathematics**

**Course No.PS02EMTH04 (Mathematical Classical Mechanics)** Total Marks: 70

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) For a conservative system, which one of the following is correct?
(a) total energy is constant (b) angular momentum is conserved
(c) linear momentum is conserved (d) none of these

(2) Motion of a particle in a plane is ___________ constraint
(a) a non-holonomic (b) a rheonomic (c) a holonomic (d) not a

(3) Degrees of freedom for a spherical pendlum is
(a) 1 (b) 0 (c) 2 (d) 6

(4) If Lagrangian does not depend on 't' explicitly then
(a) Lagrangian is conserved (b) Hamiltonian is conserved
(c) Hamiltonian is not conserved (d) energy function is conserved

(5) A coordinate $q_j$ is cyclic then,
(a) $\frac{\partial h}{\partial q_j} = 0$ (b) $\frac{\partial L}{\partial q_j} = 0$ (c) $\frac{\partial h}{\partial \dot{q}_j} = 0$ (d) $\frac{\partial L}{\partial \dot{q}_j} = 0$

(6) Which one of the following is incorrect?
(a) $\dot{\eta} = J\frac{\partial H}{\partial \eta}$ (b) $\dot{p}_j = -\frac{\partial H}{\partial q_j}$ (c) $\frac{\partial H}{\partial \dot{q}_j} = 0$ (d) none of these

(7) Which one of the following is incorrect?
(a) A symplectic matrix is invertible.
(b) Product of two symplectic matrices is also a symplectic matrix.
(c) Inverse of a symplectic matrix is also a symplectic matrix.
(d) A symplectic matrix is singular.

(8) $\{p_1, q_2\} =$ _________ ; notations being usual
(a) 0 (b) 1 (c) $p_1q_2$ (d) -1

Q-2 Answer any **Seven.** (14)

(1) State the constraints for the motion of a particle on a circle.
(2) State Lagrange's equations of motion when conservative force is present.
(3) State condition for extremum of $I = \int_{t_1}^{t_2} L(y_1, y_2, \ldots y_n, \dot{y}_1, \ldots, \dot{y}_n, x)\, dx$
(4) State the condition for conservation of generalized momentum.
(5) In usual notations show that $\frac{\partial H}{\partial t} = 0 \Rightarrow \frac{dH}{dt} = 0$.
(6) State the symplectic condition for a transformation to be canonical.
(7) State transformation equations for a generating function of type $F_3$.
(8) Linearity property for Poisson brackets.
(9) State fundamental Lagrange brackets.

Q-3

(a) State D'Alembert's principle and hence obtain equations of motion in the form $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j; j = 1,2, \dots, n$; notations being usual. (06)

(b) If $L$ is Lagrangian for a system, show that $L' = L + \frac{dF(q_1, q_2, \dots q_n, t)}{dt}$ also satisfies Lagrange's equations of motion. (06)

**OR**

(b) Giving all details obtain Lagrange's equations of motion for a simple pendulum.

Q-4

(a) State Hamilton's principle and hence derive Lagrange's equations of motion. (06)

(b) Lagrangian of a system of one degree of freedom is given by $L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{r^2}$, find energy function. Is it conserved? Justify your answer. (06)

**OR**

(b) Using calculus of variations obtain the curve for minimum surface of revolution.

Q-5

(a) Sate Hamilton's modified principle and derive Hamilton's equations of motion from it. (06)

(b) Obtain Lagrangian corresponding to the Hamiltonian $H = \frac{p_\theta^2}{2ml^2} - mgl\sin\theta$ . (06)

**OR**

(b) Discuss Routhian procedure giving a suitable example.

Q-6

(a) State and prove Jacobi identity for Poisson brackets. (06)

(b) Show that two quantities $u(q, p, t)$ and $v(q, p, t)$ are constants of motion then their Poisson bracket $[u, v]$ is also a constant of motion. (06)

**OR**

(b) Describe the method obtaining formal solution of a mechanical problem using Poisson bracket formalism.

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