[21] Seat No ——

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester II) Examination

Date: 20 – 10 – 2016 **Time: 10.00 To 1.00**

Subject: MATHEMATICS

Paper No. PS02CMTH05 – (Methods of Partial Differential Equations) Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The equation $(3D' - 2)^2(5DD' + 2D)z = 0$ has (order, degree) =

(a) (3, 1) (b) (1, 3) (c) (4, 1) (d) (4, 2)

(2) The equation $p^2 + q^2 = 0$ is same as $F(D, D')z = 0$ where $F(D, D')$ is

(a) $(D + D')^2$ (b) $(D^2 + D'^2)$ (c) $D^2D' + D'^2D$ (d) none of these

(3) Which of the following equation is reducible?

(a) $3D^2 + 2D'$ (b) D^2D' (c) $1 + DD'$ (d) $D' - 2D + D'^2$

(4) The equation $(5yD^2 - xDD' + 2y)z = 0$ is parabolic on

(a) Y-axis only (b) X-axis only (c) empty set (d) R^2

(5) Which of the following equation can be solved by general method?

(a) $2r - 3s = 0$ (b) $x^2r - y^2s = 0$ (c) $x^2r + xyt = 0$ (d) $3xr - y^2t$

(6) The equation $r + s + t = 1$ cannot be solved by

(a) General method (b) Monge's method

(c) Polynomial method (d) changing $u = \log x$, $v = \log y$

(7) The solution of Dirichlet BVP (if exists) is

(a) 0 (b) unique (c) not unique (d) none of these

(8) The one dimensional Diffusion equation is

(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} = \frac{1}{k}u_{tt}$ (c) $u_{xx} = \frac{1}{k}u_t$ (d) none of these

2. Attempt **ANY SEVEN**: [14]

(a) Define complementary function and particular integral of the equation.

(b) Eliminate functions f and g and obtain a pde: $u = f(x + iy) + g(x - iy)$.

(c) Write the equation into $\partial/\partial x, \partial/\partial y$ form: $D(2D'^2 + D'D + 3D^2)z = 0$.

(d) Find $DD'z$, if x and y in $z = z(x, y)$ is replaced by $u = \log x$ and $v = \log y$.

(e) Classify region in which equation $(2xr + yt - 2yq + xp)z = 0$ is hyperbolic.

(f) Write the Diffusion equation in cylindrical co-ordinate system.

(g) Give one examples of an irreducible equation of order 3.

(h) State Green's Identity.

(i) What is Neumann's boundary value problem?

3\. (a) If $(\beta D' + \gamma)^2$ is a factor of $F(D, D')$, then prove that $e^{-\frac{\gamma}{\beta}y}[\varphi_1(\beta x) + y\varphi_2(\beta x)]$ is a solution of $F(D, D')z = 0$, where φ_1 and φ_2 are arbitrary functions of a single variable ξ. [6]

(b) Find the general solution of **ANY ONE** of the following equations: [6]

(i) $(D^2 - D'^2 - 3D + 3D')z = x + 2y + e^{(x+2y)}$.

(ii) $(D + 3D')(2D - D' + 1)z = \cos(x - y)$.

4\. (a) Convert the equation into canonical form: $4r - y^6 t = 3y^5 q$. [6]

(b) Using Monge's method, solve **ANY ONE** of the following equations: [6]

(i) $4r - 9t = 0$. (ii) $z(qs - pt) = pq^2$.

5\. (a) Find the general solution of equation: $(x^2D^2 + 2xyDD' + xD)z = xy$. [6]

(b) Solve the Wave equation in Cartesian coordinates, by method of separation variable and show that solution is $\psi(x,y,z,t) = e^{\pm i(lx + my + nz + kct)}$ where l, m, n and k are constants with $l^2 + m^2 + n^2 = k^2$ [6]

OR

(b) By separating the variables, find the solution of three dimensional Laplace equation in cylindrical coordinate system. [6]

6\. (a) State and prove Maximum principle. [6]

(b) Solve interior Dirichlet problem for a function $\phi = \phi(r, \theta)$ for circle and show that solution is of the form $\phi(r, \theta) = \sum_{n=0}^{\infty} r^n (A_n \cos n\theta + B_n \sin n\theta)$, with A_n, B_n are constants. [6]

OR

(b) Define equipotential surfaces and show that the family of surfaces $x^2 + y^2 + z^2 = c^2$ can form an equipotential family of sufaces. [6]

x-x-x-x-x-x