(346 A-4) Seat No.:________ No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester II
Saturday, 22 October 2016
10.00 a.m. to 1.00 p.m.
PS02CMTH03 – Differential Geometry

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) A Cartesian representation of the curve $\overline{\gamma}(t) = (t, t-1)$ is

(a) $x + y = 1$ (b) $x + y = 0$ (c) $x - y = 1$ (d) $x - y = 0$

(2) Which of the following curves has exactly four vertices?

(a) $x^2 + y^2 = 1$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 9$ (d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

(3) Which of the following surfaces is not a smooth surface?

(a) cone (b) sphere (c) cylinder (d) plane

(4) The equation of the tangent plane to $x^2 + y^2 = 1$ at $(1, 0, 0)$ is

(a) $x = 0$ (b) $x^2 = 1$ (c) $x = -1$ (d) $x = 1$

(5) The Gauss map is an identity map on

(a) sphere (b) hyperboloid (c) paraboloid (d) plane

(6) The Gaussian curvature on the sphere of radius 1 is

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) 0

(7) Which of the following theorem is analogous the fundamental theorem of curve theory?

(a) Theorema Egregium (c) Gauss - Bonnet theorem
(b) Bonnet's theorem (d) None of these

(8) The geodesic curvature of the curve $\overline{\gamma}(t) = (t, -t, 0)$ on the surface $z = y^2 - x^2$ is

(a) 0 (b) 1 (c) 2 (d) 3

Q.2 Attempt any *Seven*. [14]

(a) Find a unit speed reparametrization of $\gamma(t) = (\sin 2t, \cos 2t)$.
(b) Compute the curvature of the curve $\gamma(t) = (t, t^2)$.
(c) State Wirtinger's inequality.
(d) Compute unit normal of the surface $\sigma(u, v) = (u, v, u - v)$.
(e) Define local diffeomorphism and diffeomorphism between smooth surfaces.
(f) If $f : S_1 \to S_2$ is a local isometry, then show that it is a conformal map.

(P.T.O.)

(g) Compute the Second Fundamental form on $\sigma(u,v) = (u, v, u+v)$.

(h) State the Bonnet's theorem.

(i) Show that any geodesic on a surface has a constant speed.

Q.3

(a) State and prove Isoperimetric Inequality. [6]

(b) Define reparametrization of a parametrized curve $\overline{\gamma} : (a,b) \to \mathbb{R}^n$. Show that a reparametrization of a regular curve is regular. [6]

OR

(b) Define signed curvature of a unit-speed curve $\overline{\gamma} : (a,b) \to \mathbb{R}^2$. If $k : (a,b) \to \mathbb{R}$ is a smooth map, then show that there is a unit-speed curve $\overline{\gamma} : (a,b) \to \mathbb{R}^2$ whose signed curvature is k. [6]

Q.4

(c) Define surface. Show that the set $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ is a surface. [6]

(d) Let S_1 and S_2 be two smooth surfaces. Let $f : S_1 \to S_2$ be a smooth map. If $f^*\langle v, w\rangle_p = \langle v, w\rangle_p$ for all $p \in S_1$ and $v, w \in T_pS_1$, then show that f is a local isometry. [6]

OR

(d) (i) Let $f : S_1 \to S_2$ be a local diffeomorphism, and let $\overline{\gamma}$ be a regular curve in S_1. Show that $f \circ \gamma$ is a regular curve in S_2. [3]

(ii) Compute the surface area of $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1,\ z > 0\}$. [3]

Q.5

(e) Define Weingarten map. Let $\sigma : U \to \mathbb{R}^3$ be a surface patch of an oriented surface S, and let p be in the image of σ, i.e., $p = \sigma(u,v)$ for some $(u,v) \in U$. Show that the matrix of W_p with respect to the basis $\{\sigma_u, \sigma_v\}$ of T_pS is $\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}$.

(f) Compute the principal curvatures and the principal vectors of $\sigma(u,v) = (u, v, v^2 - u^2)$ at $(0,0,0)$. [6]

OR

(f) State and prove Euler's Theorem. Hence deduce that principal curvatures at a point of a surface are maximum and minimum values of the normal curvatures and the principal vectors (directions) are the directions giving these maximum and minimum values. [6]

Q.6

(g) Determine geodesics on the unit sphere $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$. [6]

(h) Let σ be a surface patch of an oriented surface S. Then show that $L_v - M_u = L\Gamma^1_{12} + M(\Gamma^2_{12} - \Gamma^1_{11}) - N\Gamma^2_{11}$ and $M_v - N_u = L\Gamma^1_{22} + M(\Gamma^2_{22} - \Gamma^1_{12}) - N\Gamma^2_{12}$. [6]

OR

(h) State Gauss-Bonnet Theorem. Express the Christoffel's symbols Γ^1_{11} and Γ^2_{11} in terms of E, F, G and their partial derivatives. [6]