[87] SEAT No._____ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-II) Examination

March-2019

Saturday 30/03/2019

Time: 10:00AM to 01:00 PM

Subject: Mathematics **Course No.PS02EMTH22**

Mathematical Classical Mechanics

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answerfrom the options given. (08)

(1) For the motion of a particle inside a sphere constraints are_____
(a) holonomic and scleronomic (b) holonomic and rheonomic
(c) non-holonomic and scleronomic (d) non-holonomic and rheonomic

(2) Degrees of freedom for a spherical pendulum is _____ .
(a) 2 (b) 3 (c) 1 (d) 0

(3) For a particle of mass m at height h, potential energy is given by_____ .
(a) mgh (b) $-mgh$ (c) $mgl\cos\theta$ (d) none of these

(4) What are geodesics on a sphere?
(a) straight lines (b) helix (c) great circles (d) large circles

(5) $\delta\int_{t_1}^{t_2} L dt = 0$ if and only if _____.
(a) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = \frac{\partial L}{\partial q_j}$ (b) $L = 0$
(c) $L = constant$ (d) energy function vanishes

(6) Which one of the following is correct?
(a) Energy function is conserved (b) Hamiltonian is conserved
(c) $H = T + V$ (d) none of these

(7) If M is a symplectic matrix then
(a) M is singular (b) $|M| = \pm 1$ (c)$MJM^{-1} = J$ (d) $M = 0$

(8) Which one of the following is correct?
(a) $\{u, v\} = \{v, u\}$
(b) $\{u, v\} = -\{v, u\}$
(c) Lagrange bracket vanishes identically
(d) None of these

Q-2 Answer any **Seven.** (14)

(1) What is a simple harmonic oscillator? Describe constraints for it.
(2) State Lagrange's equations of motion general form.
(3) State Hamilton's principle.
(4) What is meant by brachistochrone problem?
(5) Define energy function.
(6) State Hamilton's equations of motion.
(7) What is meant by a generating function?
(8) What are fundamental Poisson brackets?
(9) Show that $\frac{du}{dt} = [u, H] + \frac{\partial u}{\partial t}$, the notations being usual.

(P.T.O.)

Q-3

(a) Obtain Lagrange's of motion when frictional forces are present. (06)

(b) Obtain Lagrangian for a double pendulum. (06)

OR

(b) In usual notations derive, $T = T_0 + T_1 + T_2$.

Q-4

(a) Discuss law of conservation of angular momentum using Lagrangian formalism. (06)

(b) Lagrangian for a system of three degrees of freedom is given by $L = \frac{I_1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 - Mgl\cos\theta$. Obtain all generalized momenta. Which of them are conserved? (06)

OR

(b) Using calculus of variations obtain geodesics in three dimensional Euclidean space.

Q-5

(a) Using Legendre transformation derive Lagrange's equations of motion from Hamilton's equations of motion. (06)

(b) Derive Hamilton's equations of motion from Hamilton's modified principle. (06)

OR

(b) State Lagrangian for a simple harmonic oscillator and hence obtain Hamilton's equations of motion for it.

Q-6

(a) State and prove Jacobi's identity for Poisson brackets. (06)

(b) Define a symplectic matrix. Show that symplectic matrices form a group. (06)

OR

(b) Determine whether the transformation $Q = \log(\sin p) - \log q, P = q\cot p$ is canonical.