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SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester II) Examination

Date: 30-03-2019, Saturday **Time: 10.00 To 1.00 p.m.**

Subject: MATHEMATICS **Paper No. PS02EMTH21 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The radius of the graph $K_{1,n}$ $(n > 1)$ is

(a) 1 (b) 2 (c) n (d) n + 1

(2) A balanced digraph is

(a) Euler (b) connected (c) symmetric (d) none of these

(3) Let T be a spanning in-tree with root R. Then

(a) $d^-(R) = 0$ (b) $d^-(R) > 0$ (c) $d^+(R) > 0$ (d) none of these

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$

(a) ne (b) 2e (c) e (d) e^2

(5) Which of the following graphs is not uniquely colourable?

(a) P_7 (b) C_7 (c) K_7 (d) $K_{3,4}$

(6) Which of the following graphs is Hamiltonian?

(a) P_{3n} (b) $K_{n,2n}$ (c) C_{3n} (d) none of these

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum ⇒ maximal (b) maximal ⇒ maximum

(c) maximum ⇒ perfect (d) maximal ⇒ perfect

(8) If $G = K_{3,n}$, then $\beta'(G) =$ ____.

(a) 3 (b) n (c) min{3, n} (d) max{3, n}

2. Attempt any SEVEN: [14]

(a) Prove: If $K_{m,n} = K_{m+n}$, then m = n = 1.

(b) Prove or disprove: A regular digraph is strongly connected.

(c) Prove or disprove: Every connected digraph has a spanning out-tree.

(d) Define fundamental circuit matrix in a digraph.

(e) Prove: If $G = C_n$ and n is odd, then $\chi(G) = 3$.

(f) Show that the condition $\kappa(G) \geq \alpha(G)$ is not necessary for a graph $G(G \neq K_2)$ to be Hamiltonian.

(g) Prove: The graph C_4 is isomorphic to $K_{2,2}$.

(h) Prove: If $S \subset V(G)$ is a vertex cover, then $V(G) - S$ is an independent set, in G.

(i) Define M-alternating and M-augmenting path (M is a matching) and give one example of each.

(P.T.O.)

3. (a) Define the following digraphs with examples: [6]
(i) Asymmetric (ii) complete asymmetric (iii) Symmetric (iv) complete symmetric.

(b) Prove that if G is connected Euler digraph, then it is balanced. [6]

OR

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \ldots n$. [6]

4. (a) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

(b) Define arborescence and prove that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is 1, −1 or 0. [6]

5. (a) Prove: If G is a simple, planer graph, then $\chi(G) \leq 5$. [6]

(b) Prove: If G is a simple graph with n vertices and $2\delta(G) \geq n \geq 3$, then G is Hamiltonian [6]

OR

(b) Find the coefficients c_3 and c_4 of chromatic polynomial of graph C_5. [6]

6. (a) Prove: If G is a bipartite graph, then $\alpha'(G) = \beta(G)$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G), \beta(G)$ and find it with the corresponding sets for $G = C_9$. [6]

x-x-x-x-x-x