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SEAT No.________ **NO. OF PRINTED PAGES: 2**

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-II) Examination
March-2019
Saturday 30/03/2019
Time: 10:00 AM to 01:00 PM

Subject: Mathematics **Course No.PS02EMTH04**

Mathematical Classical Mechanics

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) For the motion of a particle on a sphere of constant radius constraints are
(a) holonomic and scleronomic (b) holonomic and rheonomic
(c) non-holonomic and scleronomic (d) non-holonomic and rheonomic

(2) Degrees of freedom for a simple pendulum is _____ .
(a) 5 (b) 3 (c) 1 (d) 0

(3) For a particle of mass m at height h, potential energy is given by______ .
(a) mgh (b) $-mgh$ (c) $mgl\cos\theta$ (d) none of these

(4) What are geodesics on a plane?
(a) straight lines (b) helix (c) great circles (d) large circles

(5) $\int_{t_1}^{t_2} L\,dt$ is called__________
(a) Lagrangian integral (b) action integral
(c) Hamiltonian integral (d) energy integral

(6) Which one of the following is correct?
(a) Lagrangian is conserved (b) Hamiltonian is conserved
(c) $\frac{dH}{dt} = -\frac{\partial H}{\partial t}$ (d) none of these

(7) If M is Jacobian matrix for a canonical transformation then
(a) M is Identity (b) $|M| = \pm 1$ (c) $MJM^{-1} = J$ (d) M is singular

(8) Which one of the following is correct?
(a) $\{u, v\} = \{v, u\}$
(b) Lagrange bracket is canonical invariant
(c) Lagrange bracket vanishes identically
(d) Fundamental Lagrange brackets vanish identically

Q-2 Answer any **Seven.** (14)

(1) What is a simple pendulum? Describe constraints for a simple pendulum.
(2) State Lagrange's equations of motion in case of velocity dependent potential.
(3) State Hamilton's principle.
(4) What is the curve for minimum surface of revolution?
(5) Show that generalized momentum conjugate to a cyclic coordinate is conserved.
(6) State matrix form of Hamilton's equations of motion.
(7) State transformation equations for a generating function of type F_1.
(8) What are fundamental Poisson brackets?
(9) Show that $\frac{du}{dt} = [u, H] + \frac{\partial u}{\partial t}$, the notations being usual.

(P.T.O.)

Q-3

(a) Lagrangian for a system is given by L. Show that $L' = L + \frac{dF(q_1,q_2,...q_n,t)}{dt}$ also satisfies Lagrange's equations of motion. (06)

(b) Obtain Lagrange's equations of motion for a particle moving in XY-plane in plane polar coordinates. (06)

OR

(b) Obtain expression of kinetic energy in terms of generalized coordinates for a double pendulum.

Q-4

(a) Derive the condition for the extremum for $J = \int_{x_1}^{x_2} f(y, \dot{y}, x)dx$ (06)

(b) Define energy function. Explain how it is related with total energy of a system. (06)

OR

(b) Using calculus of variations solve the brachistochrone problem.

Q-5

(a) Using Legendre transformation derive Hamilton's equations of motion from Lagrange's equations of motion. (06)

(b) Discuss principle of least action. (06)

OR

(b) Lagrangian for a system is given by
$L = \frac{I_1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2$,
obtain Hamilton for the system.

Q-6

(a) State and prove Jacobi's identity for Poisson brackets. (06)

(b) For a system of one degree of freedom Hamiltonian is $H = \frac{p^2}{2} - \frac{1}{2}q^2$. Show that $u = \frac{pq}{2} - Ht$ is a constant of motion. (06)

OR

(b) Determine whether the transformation
$Q_1 = q_1$, $P_1 = p_1 - 2p_2$ $Q_2 = p_2$, $P_2 = -2q_1 - q_2$
is canonical.

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