[84]

SEAT No.________

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester II) Examination

Date: 30-03-2019, Saturday **Time: 10.00 To 1.00 p.m.**

Subject: MATHEMATICS **Paper No. PS02EMTH02 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The radius of K_n $(n > 3)$ is

(a) 1 (b) 2 (c) n (d) 0

(2) A symmetric digraph is

(a) Euler (b) connected (c) balanced (d) regular

(3) Let T be a spanning in-tree with root R. Then

(a) $d^-(R) = 0$ (b) $d^-(R) > 0$ (c) $d^+(R) > 0$ (d) none of these

(4) For $G = C_n$ with clockwise direction, rank(B) is

(a) n (b) $n - 1$ (c) 1 (d) 0

(5) The coefficient c_5 in chromatic polynomial of K_5 is

(a) 5^5 (b) 5^2 (c) 5 (d) 5!

(6) Which of the following graphs is Hamiltonian?

(a) P_{3n} (b) $K_{n, 2n}$ (c) C_{3n} (d) none of these

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximal $\Rightarrow$ perfect (b) maximum $\Rightarrow$ maximal
(c) maximum $\Rightarrow$ perfect (d) none of these

(8) If $G = K_{3,n}$, then $\alpha'(G) =$ ____.

(a) 3 (b) n (c) $\min\{3, n\}$ (d) $\max\{3, n\}$

2. Attempt any SEVEN: [14]

(a) Find the diameter of $K_{1,n}$ $(n > 1)$.
(b) Prove or disprove: An Euler digraph is connected.
(c) Give an example of a spanning in tree which is also a spanning out tree in a digraph.
(d) Define adjacency matrix in a digraph and give one example of it.
(e) Prove: If G is a bipartite graph, then $\chi(G) = 2$.
(f) What is Four color problem?
(g) Prove or disprove: The graph C_4 is isomorphic to $K_{2,2}$.
(h) Prove: If $S \subset V(G)$ is a vertex cover, then $V(G) - S$ is an independent set, in G.
(i) Define an edge cover of a graph and give one example of it.

(P.T.O.)

3. (a) Define the following with examples: [6]
(i) In-degree (ii) Out-degree (iii) Balanced digraph (iv) Regular digraph

(b) Prove that if G is a connected Euler digraph, then it is balanced. [6]

OR

(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is 1, −1 or 0. [6]

(b) Define arborescence and prove that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Define the following with examples: [6]
(i) out-tree & spanning out-tree (ii) in-tree & spanning in-tree.

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Let G be a connected graph. Prove that $\chi(G) = 2$ if and only if G does not contain an odd cycle. [6]

OR

(b) Find the coefficients c_2 and c_3 of chromatic polynomial of graph $K_{2,2}$. [6]

6. (a) Prove: If G is a bipartite graph, then $\alpha'(G) = \beta(G)$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G), \beta(G)$ and find it with the corresponding sets for $G = P_7$. [6]

x-x-x-x-x-x