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SEAT NO.________ No. of printed pages 2

Sardar Patel University

M.Sc.(Sem-II), PS02CMTH25, Methods of Partial Differential Equations;
Thursday, 28^{th} March, 2019;10.00 a.m. to 01.00 p.m.

Maximum Marks : 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Choose the most appropriate option in the following questions. [08]

1. The order of $(5D-2D')(D'-1)z=0$ is

   (a) 2 (b) 3 (c) 4 (d) None of these

2. The equation $r+2s+t=0$ is same as $F(D,D')z=0$, where $F(D,D')$ is

   (a) $D^2+2D'^2$ (b) $(D-D')^2$ (c) $(D+D')^2$ (d) None of these

3. Let $u=\log x$ and $v=\log y$ in $z=z(x,y)$. Then $x\frac{\partial^2 z}{\partial x^2}$ becomes

   (a) $\frac{1}{x}(\frac{\partial^2 z}{\partial u^2}-\frac{\partial z}{\partial u})$ (b) $\frac{\partial^2 z}{\partial u^2}-\frac{\partial z}{\partial u}$ (c) $\frac{1}{x^2}(\frac{\partial^2 z}{\partial u^2}-\frac{\partial z}{\partial u})$ (d) None of these

4. The complete integral of $z=px+qy+p^2+q^2$ is

   (a) $z=ay+bx+a+b$ (c) $z=ax+by+a^2+b^2$
   (b) $z=a^2x+b^2y+a^2+b^2$ (d) None of these

5. The equation $4y^2r+x^2t=0$ is classified as parabolic on

   (a) $x-$ axis only (b) $y-$ axis only (c) Both axes only (d) None of these

6. In Monge's method, the $\lambda-$quadratic equation of $rt-s^2+1=0$ is

   (a) λ^2-1 (b) λ^2+1 (c) λ^2+2 (d) None of these

7. The two dimensional Diffusion equation is

   (a) $u_{xx}=\frac{1}{k}u_t$ (c) $u_{xx}+u_{yy}=0$
   (b) $u_{xx}+u_{yy}=\frac{1}{c^2}u_{tt}$ (d) None of these

8. Which of the following is wave equation in spherical coordinates ?

   (a) $\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \phi^2}=0$
   (b) $\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \phi^2}=\frac{1}{k}\frac{\partial u}{\partial t}$
   (c) $\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \phi^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$
   (d) None of the above

Q.2 Attempt any seven. [14]

1. Define Comlementary function of PDE.
2. Find a PDE by eliminating f and g from $z=f(x+ay)+g(x-ay)$.
3. When is the operator $F(D,D')$ called reducible operator?
4. Find D'^2z, if x and y in $z=z(x,y)$ replace by $u=\log x$ and $v=\log y$.
5. Write necessary condition for compatibility of system of equations.

(P.T.O.)

6. Give an example of PDE which is elliptic in region $\{(x,y) \in \mathbb{R}^2 : |x| > 1\}$.

7. Classify the region in which the equation $y^2r - 3xys + 2x^2t = 0$ is hyperbolic.

8. Write Diffusion equation in spherical coordinates.

9. State Dirichlet exterior BVP for a circle.

Q.3

(a) Show that $F(D, D')\,(e^{ax+by}\phi(x,y)) = e^{ax+by}\,F(D+a, D'+b)\,\phi(x,y)$. [06]

(b) Find the general solution of $(D^2D' + D'^2 - 2)z = e^x \sin 2y$ [06]

OR

(b) Find the general solution of $(D^2 + 4DD' + 4D'^2)z = \sqrt{x - 2y}$ [06]

Q.4

(a) Find the general solution of $(x^2D^2 - y^2D'^2 + xD - yD')z = \log x$ [06]

(b) Find the general solution of $(p^2 + q^2)y = qz$ using Charpit's method. [06]

OR

(b) Find the general solution of $p^2x + q^2y = z$ using Jacobi's method. [06]

Q.5

(a) Convert $x^2r + 2xys + y^2t = 0$ into canonical form. [06]

(b) Solve $5r - 10s + 4t - rt + s^2 + 1 = 0$ using Monge's method. [06]

OR

(b) Solve $3s + rt - s^2 = 2$ using Monge's method. [06]

Q.6

(a) Solve two dimensional wave equation by method of separation variables and show that the solution can be put in the form $e^{\pm i(nx+my+kct)}$, where n, m, k are constants and $n^2 + m^2 = k^2$. [06]

(b) Solve wave equation in cylindrical coordinates by the method of separation of variables and show that the solution can be put in the form $J_m(wr)e^{\pm i(m\theta+nz+kct)}$, where $w^2+n^2 = k^2$, J_m is Bessel's function of order m. [06]

OR

(b) Find $u = u(x,y)$ such that $\nabla^2 u = 0$ in $\{(x,y) : 0 \le x \le a, 0 \le y \le b\}$ with [06]

$$u(x,0) = f(x), 0 \le x \le a$$

$$u(a,y) = 0, 0 \le y \le b$$

$$u(x,b) = 0, 0 \le x \le a$$

$$u(0,y) = 0, 0 \le y \le b$$

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