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SARDAR PATEL UNIVERSITY

M.Sc. (Semester - II) Examination
Monday April 23, 2018
Time: 10:00 a.m. to 01:00 p.m.
Subject: Mathematics
Course No. PS02EMTH22 (Mathematical Classical Mechanics) Total Marks : 70

Note: (1) All the questions are to be answered in the answer book only.
(2) Figures to the right indicate marks of the respective question.
(3) Assume standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions: (08)

1. The two constraints of a particle moving inside a circle are ______.
   (a) holonomic and rheonomic (c) holonomic and non-holonomic
   (b) non-holonomic and rheonomic (d) both holonomic
2. The degrees of freedom of a rigid body with 12 particles in space is ______.
   (a) 6 (b) 12 (c) 30 (d) 36
3. The curve between two points $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ such that $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ is minimum is a ______.
   (a) great circle (b) straight line (c) catenary (d) cycloid
4. The energy function for a system with scleronomic constraints and potential independent of generalized velocities is given by $h =$ ______.
   (a) L (b) $-L$ (c) $L - 2V$ (d) $L + 2V$
5. If Hamiltonian H does not depend on t explicitly, then ______ is conserved.
   (a) Lagrangian (b) Routhian (c) energy function (d) total energy
6. If all the coordinates in a system are cyclic, then the Routhian $R =$ ______.
   (a) H (b) $-H$ (c) L (d) $-L$
7. If M is a symplectic matrix M, then the matrix ______ need not be symplectic.
   (a) M^2 (b) $2M$ (c) M' (d) $-M$
8. $[p_2 + q_1, p_1] =$ ______; notations being usual.
   (a) $p_2p_1 + q_1p_1$ (b) 0 (c) -1 (d) 1

Q-2 Answer *any seven* of the following: (14)

(a) State the principle of virtual work.

(b) State Lagrange's equations of motion of a system with conservative forces and velocity independent potential.

(c) What is the condition for extremum of the integral $\int_{x_1}^{x_2} g(y, \dot{y}, x)\, dx$?

(d) Show that momentum conjugate to a cyclic coordinate is conserved.

(e) Define Routhian of a system.

(f) State Hamilton's modified principle.

(g) For a system of n-degrees of freedom, define phase space and canonical variables.

[P.T.O.]

(h) Using Poisson brackets show that $\frac{\partial H}{\partial t} = \frac{dH}{dt}$, where H is Hamiltonian of a system.

(i) State the transformation equations for a canonical transformation with generating function of type F_2.

Q-3 (a) Using D'Alembert's principle, derive the general form of Lagrange's equations of motion. (06)

(b) In the following systems, describe and classify the constraints, hence determine degrees of freedom. (06)

1. Two particles moving in space connected by an in-extensible rod of length l.
2. Motion of a particle on an ellipse.

OR

(b) Describe simple pendulum and derive its Lagrange's equations of motion. (06)

Q-4 (a) Describe Brachistochrone problem and obtain its solution. (06)

(b) State Hamilton's principle and derive Lagrange's equations of motion from it. (06)

OR

(b) Lagrangian of a system is given by $L = \frac{m}{2}l^2(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + mgl\cos\theta$. How many generalized coordinates are there? Which of them are cyclic? Compute the energy function. Is it conserved? Justify. (06)

Q-5 (a) Derive Lagrange's equations of motion from Hamilton's equations of motion. (06)

(b) Let $H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$ be Hamiltonian of a system. Construct the corresponding Lagrangian and hence derive Lagrange's equations of motion. (06)

OR

(b) Consider a Lagrangian of the form $L = \frac{1}{2}m(\dot{x}^2 - \omega^2 x^2)e^{\gamma t}$, where m is the mass, t is time, and ω and γ are positive constants. Construct the Hamiltonian and derive Hamilton's equations of motion. Is Hamiltonian a constant of motion? Justify. (06)

Q-6 (a) State and prove Poisson's theorem. (06)

(b) Define canonical transformation. Check whether the following transformations are canonical: (06)

1. $Q = \frac{1}{2}(q^2 + p^2)$, $P = -\tan^{-1}\left(\frac{q}{p}\right)$.
2. $Q = q^2\cos 2p$, $P = q^2\sin 2p$.

OR

(b) Define a symplectic matrix. Show that the set of all $2n \times 2n$ symplectic matrices forms a group under usual matrix multiplication. (06)

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