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SEAT No.______

[14]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester II) Examination

Date: 23-04-2018 **Time: 10.00 To 1.00 p.m.**

Subject: MATHEMATICS **Paper No. PS02EMTH21 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The diameter of the graph $K_{1,n}$ $(n > 1)$ is

(a) 1 (b) 2 (c) n (d) n + 1

(2) A symmetric digraph is

(a) Euler (b) connected (c) balanced (d) none of these

(3) For $G = C_n$ with clockwise direction, rank(B) is

(a) 1 (b) n – 1 (c) n (d) none of these

(4) If G is a complete symmetric digraph with n vertices, then $|E(G)| =$

(a) $\frac{n(n-1)}{2}$ (b) n (c) $n(n-1)$ (d) n^2

(5) The coefficient c_5 in chromatic polynomial of K_5 is

(a) 5^5 (b) 5^2 (c) 5 (d) 5!

(6) Which of the following graphs is Hamiltonian?

(a) P_n (b) P_{2n} (c) C_{2n} (d) $K_{n,2n}$

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum

(b) perfect $\Rightarrow$ maximal (d) maximal $\Rightarrow$ perfect

(8) If $G = K_{3,n}$, then $\beta(G) =$ ____.

(a) 3 (b) n (c) $\min\{3, n\}$ (d) $\max\{3, n\}$

2. Attempt any SEVEN: [14]

(a) Prove: If $K_{m,n} = K_{m+n}$, then $m = n = 1$.

(b) Prove or disprove: A balanced digraph is regular.

(c) Define adjacency matrix in a digraph and give one example of it.

(d) Give an example of a spanning in tree which is also a spanning out tree in a digraph.

(e) Prove: If G is a bipartite graph, then $\chi(G) = 2$.

(f) What is Four color problem?

(g) Prove or disprove: The graph C_4 is isomorphic to $K_{2,2}$.

(h) Prove: If $S \subset V(G)$ is a vertex cover, then $V(G) - S$ is an independent set, in G.

(i) State Hall's theorem.

[P.T.O.]

3. (a) Define the following digraphs with examples: [6]
(i) Asymmetric (ii) complete asymmetric (iii) Symmetric (iv) complete symmetric.

(b) Prove that if G is connected Euler digraph, then it is balanced. [6]

OR

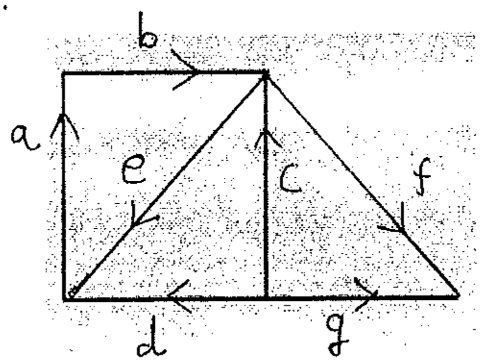
(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Define arborescence and prove that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

(b) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

OR

(b) Define a fundamental circuit matrix in a digraph and find it w. r. t. spanning tree $T = \{a, d, e, g\}$ in digraph below: [6]



5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Let G be a k-chromatic graph with n vertices. Prove that $n \leq k\,\alpha(G)$. [6]

OR

(b) Find the coefficients c_2 and c_3 of Chromatic polynomial of graph $K_{1,3}$. [6]

6. (a) Prove: A matching M in a graph G is maximum if and only if G has no M-augmenting path. [6]

(b) Let G be a graph (without isolated vertex) with n vertices. Then prove that $\alpha'(G) + \beta'(G) \leq n$. [6]

OR

(b) Define maximum matching and perfect matching and find it for $G = P_6$. [6]

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