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SEAT No.________ NO. OF PRINTED PAGES: 2

[13/A-18/A-19]

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-II) Examination

April – 2018

Monday 23/04/2018

Time: 10:00 AM to 1:00 PM

Subject: Mathematics Total Marks: 70

Course No.PS02EMTH04 (Mathematical Classical Mechanics)

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) For a conservative system, which one of the following is correct?
(a) total energy is zero (b) angular momentum is zero
(c) linear momentum is zero (d) none of these

(2) Motion of a particle in the unit sphere is ___________ constraint
(a) a non-holonomic (b) a rheonomic (c) a holonomic (d) not a

(3) Degrees of freedom for a Atwood's machine is
(a) 1 (b) 0 (c) 2 (d) 6

(4) The condition for extremum of $J = \int_{x_1}^{x_2} f(y, \dot{y}, x)dx$ is _______
(a) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$ (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial x} = 0$
(c) $\frac{d}{dy}\left(\frac{\partial f}{\partial \dot{x}}\right) - \frac{\partial f}{\partial x} = 0$ (d) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial y} = 0$

(5) A coordinate q_j is said be cyclic if,
(a) $\frac{\partial h}{\partial q_j} = 0$ (b) $\frac{\partial L}{\partial q_j} = 0$ (c) $\frac{\partial h}{\partial \dot{q}_j} = 0$ (d) $\frac{\partial L}{\partial \dot{q}_j} = 0$

(6) Which one of the following is correct?
(a) A symplectic matrix is invertible.
(b) Identity matrix is not symplectic.
(c) Canonical transformations are not invertible.
(d) Product of two symplectic matrices is not symplectic.

(7) Pick up the incorrect statement:
(a) Fundamental Poisson brackets are invariant under a canonical transformation.
(b) The value of a fundamental Poisson bracket is constant.
(c) Fundamental Poisson brackets follow Jacobi's identity.
(d) None of these.

(8) $\{p_1, q_2\} =$ _________ ; notations being usual
(a) 0 (b) 1 (c) $p_1 q_2$ (d) -1

Q-2 Answer any **Seven.** (14)

(1) Obtain degrees of freedom of simple pendulum.
(2) State principle of virtual work.
(3) State condition for extremum of $I = \int_{t_1}^{t_2} L(q_1, q_2, \dots q_n, \dot{q}_1, \dots, \dot{q}_n, t)\, dt$
(4) Show that generalized momentum conjugate to a cyclic coordinate is conserved.
(5) Show that a coordinate cyclic in Lagrangian is also cyclic in Hamiltonian.

[P.T.O.]

(6) Define a symplectic matrix.

(7) State transformation equations for a generating function of type F_4.

(8) State matrix form of Poisson brackets.

(9) State Jacobi's identity for Poisson brackets.

Q-3

(a) State Lagrange's equations of motion in general form and derive the form when frictional forces are present. (06)

(b) Giving all details obtain Lagrange's equations of motion for a simple pendulum. (06)

OR

(b) Describe simple harmonic oscillator and obtain Lagrange's equation of motion for a simple harmonic oscillator.

Q-4

(a) Derive condition for extremum of $J = \int_{x_1}^{x_2} f(y, \dot{y}, x)dx$. (06)

(b) Derive law of conservation for linear momentum using Lagrangian formalism. (06)

OR

(b) Using calculus of variations obtain the curve of shortest distance between two points in the space.

Q-5

(a) Describe Legendre transformation and derive Hamilton's equations of motion from it. (06)

(b) Describe Routhian procedure giving an example. (06)

OR

(b) Obtain Hamiltonian corresponding to the Lagrangian

$$L = \frac{m}{2}(\dot{q}^2 \sin^2 \omega t + \dot{q} q \sin 2\,\omega t + q^2\omega^2)$$

Also obtain Hamilton's equations of motion.

Q-6

(a) Define Poisson brackets. Show that they are invariant under a canonical transformation. (06)

(b) Show that Poisson bracket of two constants of motion is also a constant of motion. (06)

OR

(b) Hamiltonian for a system is given by $H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$. Using Poisson bracket formalism obtain the expression of q and p as functions of t.

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