SEAT No.______

[12]

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SARDAR PATEL UNIVERSITY
M. Sc. (Semester II) Examination

Date: 23-04-2018 **Time: 10.00 To 1.00 p.m.**

Subject: MATHEMATICS **Paper No. PS02EMTH02 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) For $G = K_n$ $(n > 2)$, if diam(G) = d and rad(G) = r, then
(a) $d = r$ (b) $d < r$ (c) $d > r$ (d) none of these

(2) A balanced digraph is
(a) Euler (b) symmetric (c) connected (d) none of these

(3) For $G = C_n$ with clockwise direction, rank(B) is
(a) n (b) $n - 1$ (c) 1 (d) none of these

(4) Let T be a spanning out-tree with root R. Then
(a) $d^+(R) = 0, d^-(R) = 0$ (c) $d^+(R) > 0, d^-(R) > 0$
(b) $d^+(R) = 0, d^-(R) > 0$ (d) $d^+(R) > 0, d^-(R) = 0$

(5) The coefficient c_5 in chromatic polynomial of K_5 is
(a) 5^5 (b) 5^2 (c) 5 (d) 5!

(6) Which of the following graphs is not Hamiltonian?
(a) $K_{n, n}$ (b) K_n (c) P_n (d) C_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum
(b) maximum $\Rightarrow$ maximal (d) maximal $\Rightarrow$ perfect

(8) If $G = K_{3, n}$, then $\alpha(G) =$ ____.
(a) max{3, n} (b) min{3, n} (c) 3 (d) n

2. Attempt any SEVEN: [14]

(a) Prove: If $K_{m, n} = K_{m+n}$, then $m = n = 1$.
(b) Prove or disprove: An Euler digraph is connected.
(c) Define fundamental circuit matrix in a digraph.
(d) Define spanning in tree and give one example of it.
(e) Prove: If $\chi(G) = 2$, then G is a bipartite graph.
(f) Is K_5 uniquely colourable? Why?
(g) Prove: The graph C_4 is isomorphic to $K_{2, 2}$.
(h) Prove: If $S \subset V(G)$ is an independent set, then $V(G) - S$ is a vertex cover, in G.
(i) Define perfect matching and find one perfect matching in $K_{3, 3}$.

[P.T.O.]

3. (a) Define the following with examples: [6]
(i) In-degree (ii) Out-degree (iii) Balanced digraph (iv) Regular digraph

(b) Prove that if G is connected Euler digraph, then it is balanced. [6]

OR

(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

(b) Prove that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \dots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \dots n$. [6]

5. (a) Prove: If G is simple graph with n vertices and $2\delta(G) \geq n \geq 3$, then G is Hamiltonian. [6]

(b) Let G be a k-chromatic graph with n vertices. Prove that $n \leq k\,\alpha(G)$. [6]

OR

(b) Find the coefficients c_2 and c_3 of Chromatic polynomial of graph $K_{2,2}$. [6]

6. (a) Let G be a graph (no isolated vertex) with n vertices. Then prove that $\alpha'(G) + \beta'(G) = n$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha'(G)$, $\beta(G)$ and find it with the corresponding sets for $G = P_5$. [6]

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