SEAT No.________

[26/A-23]

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Sardar Patel University

M.Sc. (Sem-II), PS02CMTH25, Methods of Partial Differential Equations;
Friday, 20^{th} April, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of $(D+D')(D'-2)^2z=0$ is
   (A) 2 (B) 3 (C) 4 (D) 1
2. The equation $r-4q-t=0$ can be written in the form $F(D,D')z=0$, where $F(D,D')$ equals
   (A) $D^2-4D-D'^2$ (B) $D^2-4D'-D'^2$
   (C) $D'^2-4D'-D^2$ (D) D'^2-4D-D^2
3. The complete integral of $pqz=p^2(xq+p^2)+q^2(yp+q^2)$ is
   (A) $abz=a^2bx+ab^2y+a^4+b^4$ (B) $z=a^2bx+ab^2y+a^4+b^4$
   (C) $abz=a^2bx+ab^2y+a^3+b^3$ (D) none of these
4. Let $u=\log x$ and $v=\log y$ in $z=z(x,y)$. Then $x\frac{\partial^2 z}{\partial x^2}$ becomes
   (A) $\frac{1}{x}\left(\frac{\partial^2 z}{\partial u^2}-\frac{\partial z}{\partial u}\right)$ (B) $\frac{\partial^2 z}{\partial u^2}-\frac{\partial z}{\partial u}$
   (C) $\frac{1}{x^2}\left(\frac{\partial^2 z}{\partial u^2}-\frac{\partial z}{\partial u}\right)$ (D) none of these
5. The equation $x^2r-2s+t=0$ is elliptic, if
   (A) $|x|=1$ (B) $|x|<1$ (C) $|x|>1$ (D) none of these
6. In Monge's method, the $\lambda-$ quadratic equation of $3r+4s+t+rt-s^2=1$ is
   (A) $(2\lambda-1)^2$ (B) $(\lambda+2)^2$ (C) $(\lambda-2)^2$ (D) $(2\lambda+1)^2$
7. The solution of $x^2y''+xy'+(n^2x^2-m^2)y=0$ is
   (A) $J_m(x)$ (B) $J_n(mx)$ (C) $J_m(nx)$ (D) $J_n(x)$
8. The three dimensional wave equation is
   (A) $u_{xx}+u_{yy}+u_{zz}=0$ (B) $u_{xx}+u_{yy}+u_{zz}=\frac{1}{c^2}u_t$
   (C) $u_{xx}+u_{yy}=\frac{1}{c^2}u_{tt}$ (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define complementary function of pde.
(b) Find a pde by eliminating f and g from $z=f(x^2-y)+g(x^2+y)$.
(c) Give an example of pde whose general solution is $\phi_1(x+2y)+\phi_2(x-2y)$, where ϕ_1 and ϕ_2 are arbitrary functions.
(d) Find D'^3z, if x and y in $z=z(x,y)$ replaced by $u=\log x$ and $v=\log y$.
(e) Show that $p=F(x,y)$ and $q=G(x,y)$ are compatible if $\frac{\partial F}{\partial y}=\frac{\partial G}{\partial x}$.
(f) Give an example of pde which is hyperbolic in region $\{(x,y)\in\mathbb{R}^2:|x|>2\}$.
(g) Find $u=u(x,y)$ and $v=v(x,y)$ to covert $r+2s+t=0$ in the canonical form.
(h) Write wave equation in spherical coordinates.
(i) State Dirichlet interior BVP for a circle.

(P. T. O.)

Q.3

(a) If $(\alpha D + \beta D' + \gamma)^2$ is a factor of $F(D, D')$ with $\alpha \neq 0$, then prove that $e^{-\frac{\gamma}{\alpha}x}[\phi_1(\beta x - \alpha y) + x\phi_2(\beta x - \alpha y)]$ is a solution of $F(D, D')z = 0$, where ϕ_1 and ϕ_2 are arbitrary functions. [6]

(b) Find the general solution of $(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x + y)$. [6]

OR

(b) Find the general solution of $(D^2 + 2DD' - 8D'^2)z = \sqrt{2x + 3y}$.

Q.4

(a) Find the general solution of $(x^2D^2 - y^2D'^2 - yD' + xD)z = \log(\frac{x}{y})$. [6]

(b) Find the complete integral of $xy(p - q) = y - x$ using Charpit's method. [6]

OR

(b) Find the complete integral of $p^2x + q^2y = z$ using Jacobi's method.

Q.5

(a) Convert $r + 2s + 5t + p - 2q = 3z$ into the canonical form. [6]

(b) Solve $q^2r + p^2t = 2pqs$ using Monge's method. [6]

OR

(b) Solve $rt - s^2 + 1 = 0$ using Monge's method.

Q.6

(a) Solve two dimensional heat equation by the method of separation of variables and show that the solution can be put in the form $e^{i(nx+my)-(n^2+m^2)kt}$, where n, m, k are constants. [6]

(b) Solve Laplace equation in cylindrical coordinates by the method of separation of variables and show that the solution can be put in the form $J_n(mr)e^{mz+in\theta}$, where n, m are constants and J_n is Bessel's function of order n. [6]

OR

(b) Find $u = u(x, y)$ such that $\nabla^2 u = 0$ in $\{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$ with

$$\begin{aligned} u(x,0) &= f(x), \; 0 \leq x \leq a \\ u(a,y) &= 0, \quad 0 \leq y \leq b \\ u(x,b) &= 0, \quad 0 \leq x \leq a \\ u(0,y) &= 0, \quad 0 \leq y \leq b. \end{aligned}$$

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