Sc

SEAT No.________

[25/A-22]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-II), PS02CMTH05, Methods of Partial Differential Equations;
Friday, 20^{th} April, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of $(D+1)(D'-D)^2z=0$ is
   (A) 2 (B) 3 (C) 4 (D) 1
2. The equation $r-2q-t=0$ can be written in the form $F(D,D')z=0$, where $F(D,D')$ equals
   (A) $D^2-2D'-D'^2$ (B) $D^2-2D-D'^2$
   (C) $D'^2-2D'-D^2$ (D) D'^2+2D-D^2
3. The equation $4y^2r+x^2t=0$ is classified as parabolic on
   (A) x-axis only (B) y-axis only
   (C) both axes only (D) none of these
4. In Monge's method, the $\lambda-$ quadratic equation of $3s+rt-s^2-2=0$ is
   (A) $2\lambda^2-3\lambda+1=0$ (B) $2\lambda^2+3\lambda+1=0$
   (C) $2\lambda^2+1=0$ (D) none of these
5. The solution of $x^2y''+xy'+(x^2-m^2)y=0$ is
   (A) $J_m(x)$ (B) $J_n(mx)$ (C) $J_m(nx)$ (D) $J_n(x)$
6. Which one from the following is Laplace equation ?
   (A) $u_{xx}+u_{yy}+u_{zz}=0$ (B) $u_{xx}+u_{yy}=\frac{1}{c^2}u_{tt}$
   (C) $u_{xx}=\frac{1}{k}u_t$ (D) none of these
7. A solution of $---$ is known as equipotential function.
   (A) Laplace equation (B) heat equation
   (C) wave equation (D) none of these
8. If u_1 and u_2 are any two solutions of Dirichlet BVP, then
   (A) $u_1=\alpha u_2$ $(1\neq\alpha\in\mathbb{R})$ (B) $u_1-u_2=\alpha$ $(0\neq\alpha\in\mathbb{R})$
   (C) $u_1=u_2$ (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define general solution of partial differential equation.
(b) Find a pde by eliminating f and g from $z=f(x-y)+g(x+y)$.
(c) Solve: $(D^2-D')z=0$.
(d) Find D^2z, if x and y in $z=z(x,y)$ replaced by $u=\log x$ and $v=\log y$.
(e) Classify the region in which the equation $r-2s+t+2p-q=0$ is parabolic.
(f) Give an example of pde which is hyperbolic in region $\{(x,y)\in\mathbb{R}^2:|x|>1\}$.
(g) Find $u=u(x,y)$ and $v=v(x,y)$ to covert $r-t=0$ in the canonical form.
(h) State minimum principle.
(i) State Harnack's theorem.

(P.T.O.)

Q.3

(a) If $\alpha D + \beta D' + \gamma$ is a factor of $F(D, D')$ with $\alpha \neq 0$, then prove that $e^{-\frac{\gamma}{\alpha}x}\phi(\beta x - \alpha y)$ is a solution of $F(D, D')z = 0$, where ϕ is arbitrary function. [6]

(b) Find particular integral of $(D^2 - D'^2 - 3)z = e^{2x+y}$. [6]

OR

(b) Find the general solution of $(D^2 - D'^2)z = \cos(2x - 3y)$.

Q.4

(a) Convert $r - x^2t = 0$ into the canonical form. [6]

(b) Solve $r - 4t = 0$ using Monge's method. [6]

OR

(b) Solve $3r + 4s + t + rt - s^2 - 1 = 0$ using Monge's method.

Q.5

(a) Find the general solution of $(x^2D^2 - y^2D'^2 - yD' + xD)z = xy^2$. [6]

(b) Solve $\frac{\partial^2\varphi}{\partial x^2} = \frac{1}{k}\frac{\partial\varphi}{\partial t}$ by the method of separation of variables and show that the solution can be put in the form $\varphi(x, t) = e^{inx - kn^2t}$, where n is a constant. [6]

OR

(b) Derive Laplace equation in cylindrical coordinates.

Q.6

(a) State and prove maximum principle. [6]

(b) Discuss Dirichlet exterior BVP for a circle. [6]

OR

(b) Show that the solution of Neumann BVP is unique upto additive constant.

——X——