SEAT No.________ NO. OF PRINTED PAGES: 2

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[38 & A-26]

SARDAR PATEL UNIVERSITY

M.Sc. (Semester-II) Examination
April-2018
Friday, April 13, 2018
Time: 10:00 AM to 01:00 PM

Subject: Mathematics **Course No.PS02CMTH03**

Differential Geometry

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) A Cartesian representation of the curve $\bar{\gamma}(t) = (t, t)$
(a) $x + y = 1$ (b) $x - y = 1$ (c) $x - y = 0$ (d) none of these

(2) Let $\bar{\gamma}(t)$ be a regular curve then
(a) it is not parametrized.
(b) it has no unit-speed reparametrization.
(c) it has unit-speed reparametrization.
(d) it's curvature vanishes everywhere.

(3) Which of the following represents a circle?
(a) $(b \sin t,\ b \cos t,\ c)$ (b) $(\sinh t, \cosh t)$
(c) $(b, a \sin t, b \cos t)$ (d) none of these

(4) Which of the following can not be covered with a single patch?
(a) sphere (b) cylinder (c) ellipsoid (d) none of these

(5) Which one of the following is not correct?
(a) The derivative is a linear map.
(b) Derivative of composition of maps is composition of derivatives.
(c) Derivative of product of two maps is product of derivatives.
(d) Derivative of addition of two maps is addition of derivatives.

(6) The equation of the tangent plane to the unit sphere at $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ is
(a) $x + y + z = 0$ (b) $x + y + z = 1$
(c) $x + y + z = \sqrt{3}$ (d) $x + y + z = \frac{1}{\sqrt{3}}$

(7) The point (0,0,0) on the surface $x = y^2 + z^2$ is
(a) parabolic (b) elliptic (c) hyperbolic (d) flat

(8) The sum of interior angles of a triangle on a sphere is
(a) $> \pi$ (b) $< \pi$ (c) $= \pi/2$ (d) $= \pi$

Q-2 Answer any **Seven.** (14)

(1) Compute signed unit normal to the curve $\bar{\gamma}(t) = (e^{kt} \cos t, e^{kt} \sin t)$.
(2) Compute arc-length of the curve $\bar{\gamma}(t) = (\cos 2t, \sin 2t)$ between the points $\bar{\gamma}(\pi/2)$ and $\bar{\gamma}(3\pi/2)$.
(3) Define turning angle of a unit-speed curve.
(4) For real numbers $a, b, c, d; c \neq 0$ define a surface patch on the set $S = \{(x, y, z) \in R^3 : ax + by + cz = d\}$.
(5) For smooth surfaces let $f: S_1 \to S_2$ be a local isometry. If $\bar{\gamma}$ is a unit-speed curve in S_1 then show that $f o \bar{\gamma}$ is a unit-speed curve in S_2.

(P.T.O.)

(6) State the formula for finding surface area in terms of first fundamental magnitudes.

(7) State the expression of mean curvature of a surface at a point in terms of second order magnitudes.

(8) Let U be a connected subset of R^2. If both the principal curvatures at every point of $\sigma(U)$ are zero then show that $\sigma(U)$ is a part of a plane

(9) State Gauss' theorema egregium.

Q-3

(a) Let $\bar{\gamma}$ be a unit-speed curve in R^3 such that all the tangents to $\bar{\gamma}$ pass through a fixed point. Show that $\bar{\gamma}$ is part of a line. (06)

(b) Show that the length and the area of interior of a simple closed curve are invariant under a direct isometry. (06)

OR

(b) Compute curvature and torsion of the curve
$\bar{\gamma}(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{1}{\sqrt{2}}\right)$

Q-4

(a) Define tangent vector and tangent space to a surface at a point on it. Show that the tangent space at a point $p \in S$ is a subspace of R^3. What are its the basis vectors? (06)

(b) Let $f: S_1 \to S_2$ and $g: S_2 \to S_3$ be smooth maps. Show that $D_p(gof) = (D_{f(p)}g)oD_pf$ for all $p \in S_1$. (06)

OR

(b) Compute the first fundamental form on the surface
$\sigma(u,v) = (\cosh u, \sinh u, v)$

Q-5

(a) Let p be a point on an oriented surface S. Show that there are real numbers k_1, k_2 and a basis $\{\bar{t}_1, \bar{t}_2\}$ of T_pS such that $W_p(\bar{t}_i) = k_i\bar{t}_i$ for $i = 1,2$. When the set $\{\bar{t}_1, \bar{t}_2\}$ is orthogonal? (06)

(b) Compute the Gaussian curvature and mean curvature of the surface $z = f(x,y)$. (06)

OR

(b) Compute principal curvatures of the surface $\sigma(u,v) = (u, v, v^2 - u^2)$.

Q-6

(a) Define geodesic on a surface and show that any geodesic has a constant speed. (06)

(b) Show that the Gaussian curvature of a surface is preserved under local isometries. (06)

OR

(b) Obtain geodesics on a sphere.

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