SEAT No.________

[37/A-25]

No of printed pages: 2

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M.Sc. (Sem-II), PS02CMTH02, Algebra-I;

Wednesday, 11^{th} April, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The number of elements in a field can be
   (A) 6 (B) 15 (C) 4 (D) 36
2. Which is from the following is not an Euclidean ring ?
   (A) $\mathbb{C}[x]$ (B) $(\mathbb{Z}, +, \cdot)$ (C) $(2\mathbb{Z}, +, \cdot)$ (D) none of these
3. The polynomial $x^2 - 5$ is reducible over
   (A) $\mathbb{Q}$ (B) $\mathbb{Z}$ (C) $\mathbb{C}$ (D) none of these
4. Which is unit in $J[i]$?
   (A) $2i$ (B) $1 + i$ (C) $-i$ (D) $-2i$
5. The content of a polynomial $4x^2 + 2x - 2$ is
   (A) 2 (B) 4 (C) 8 (D) 1
6. $[\mathbb{C} : \mathbb{R}] =$
   (A) 1 (B) 2 (C) 3 (D) ∞
7. Which one from the following is not a radical extension of $\mathbb{Q}$?
   (A) $\mathbb{Q}(i)$ (B) $\mathbb{Q}(\sqrt{2})$ (C) $\mathbb{Q}(\pi)$ (D) $\mathbb{Q}(\sqrt{2}, \sqrt{3})$
8. The group S_n is solvable for $n =$
   (A) 5 (B) 6 (C) 7 (D) 4

Q.2 Attempt any ***seven***: [14]

(a) Show that every field is Euclidean ring.
(b) State Wilson's theorem.
(c) Show that the only units in $F[x]$ are constant polynomials.
(d) Define primitive polynomial and give one example of it.
(e) $f(x) \in F[x]$ and $0 \neq a \in F$. If $f(ax)$ is irreducible over F then show that $f(x)$ is irreducible over F.
(f) Find the splitting field of $(x^2 - 7)(x^2 - 4)$ over $\mathbb{Q}$.
(g) Is $\sqrt{2} + \sqrt{3}$ algebraic over $\mathbb{Q}$? Justify.
(h) Define normal extension and give one example of it.
(i) Prove or disprove: Every cyclic group is solvable.

Q.3

(a) Show that every Euclidean ring is a principal ideal ring and possesses a unit element. [6]
(b) State and prove Fermat's theorem. State results which you use. [6]

OR

(b) In a Euclidean ring R, show that a non-zero element a in R is a unit if and only if $d(a) = d(1)$.

(P.T.O.)

Q.4

(a) State and prove Eisenstein criterion. [6]

(b) Prove that the product of any two primitive polynomials is a primitive polynomial. [6]

OR

(b) Show that $15x^4 - 10x^2 + 9x + 21$ is irreducible over $\mathbb{Q}$.

Q.5

(a) If K is an extension of F and $a, b \in K$ are algebraic over F, then show that $a + b$ is algebraic over F. [6]

(b) If L is algebraic extension of K and if K is algebraic extension of F, then show that L is algebraic extension of F. [6]

OR

(b) Let $f(x) \in F[x]$. Then show that $f(x)$ has a multiple root iff $f(x)$ and $f'(x)$ have nontrivial common factor.

Q.6

(a) Show that K is a normal extension of F if K is the splitting field of some polynomial over F. [6]

(b) Show that the group S_n, $n \geq 5$ is not solvable. [6]

OR

(b) State the fundamental theorem of Galois theory.

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