SEAT No.________

No of printed pages: 2

[62 & A-10]

Sardar Patel University

Mathematics

M.Sc. Semester II

Thursday, 02 November 2017

2.00 p.m. to 5.00 p.m.

PS02CMTH01 - Real Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) Let $A = (-1, 1)$ and $B = [-1, 1]$. Which of the following is true?

(a) $m^*A < m^*B$ (b) $m^*A > m^*B$ (c) $m^*A = m^*B$ (d) none of these

(2) The Lebesgue integral of the constant function 1 over $[-1, 1] \cap (\mathbb{R} - \mathbb{Q})$ is ____

(a) 1 (b) 2 (c) 4 (d) ∞

(3) Let $f : [0, 1] \to \mathbb{R}$ be defined as $f(x) = 1$ if $x \in \mathbb{R} - \mathbb{Q}$ and $f(x) = 0$ if $x \in \mathbb{Q}$. The the value of the lower Riemann integral $\underline{\int_0^1} f(x)dx$ is ____

(a) 0 (b) 1 (c) 2 (d) ∞

(4) The value of $\lim\limits_{n\to\infty} \int\limits_{[0,1]} x^n dx$ is ____

(a) 0 (b) 1 (c) ∞ (d) none of these

(5) Let $f(x) = \sin x$ for $x \in [0, \pi]$. Then f^- is ____

(a) $-|\sin x|$ (b) $\sin x$ (c) 0 (d) $-\sin x$

(6) Let $f_n(x) = 2 - \frac{1}{n}$ for all $x \in \mathbb{R}$, and let $f = \lim_{n\to\infty} f_n$. Then $\int_{\mathbb{R}} f$ is ____

(a) 1 (b) 2 (c) ∞ (d) none of these

(7) Let $F(x) = \int_{-1}^{x} |t|dt$ for all $x \in [-1, 1]$. Which of the following is not true?

(a) F is absolutely continuous
(b) F is of bounded variation
(c) F is differentiable
(d) none of these

(8) Let $f(x) = x^2$. Then $T_3^4(f)$ is

(a) 9 (b) 1 (c) 3 (d) 16

Q.2 Attempt any *Seven*. [14]

(a) Show union of two σ- algebras need not be a σ- algebra.

(b) Find the measure of the set $\{\pi + x : x \in \mathbb{Q}\}$.

(c) If f^2 is measurable over $\mathbb{R}$, then show that f need not be measurable.

(d) If f is a nonnegative measurable function over a measurable set E and if $\int_E f = 0$, then show that $f = 0$ a.e. on E.

(e) Give an example of a Lebesgue integrable function over $[0, 1]$ which is not Riemann integrable.

(f) If f is integrable over E and if A and B are disjoint measurable subsets of E, then show that $\int_{A\cup B} f = \int_A f + \int_B f$.

(g) If f is integrable over E, then show that $|\int_E f| \leq \int_E |f|$.

(h) State Jordan's Lemma. Write $-\sin x$ as a difference of monotonic functions over $[0, \frac{\pi}{2}]$.

(i) If f is a continuous function on $[0,1]$, then show that $F(x) = \int_0^x f(t)dt$, $x \in [0,1]$, is a bounded function.

Q.3

(a) If $a \in \mathbb{R}$, then show that $m^*(a, \infty) = \infty$. [6]

(b) Let E be a measurable set, and let $\{f_n\}$ be a sequence of measurable functions. Prove that $\sup_n f_n$ and $\inf_n f_n$ are measurable functions. Deduce that $\liminf_n f_n$ and $\limsup_n f_n$ are measurable functions. [6]

OR

(b) Prove that there exists a nonmeasurable set. [6]

Q.4

(c) Let E be a measurable set of finite measure, and $\{f_n\}$ a sequence of measurable functions defined on E. Let f be a real valued function such that for each x in E we have $f_n(x) \to f(x)$. Show that given $\epsilon > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ with $mA < \delta$ and an integer N such that for all $x \notin A$ and all $n > N$, $|f_n(x) - f(x)| < \epsilon$. [6]

(d) Let f and g be nonnegative measurable functions on a measurable set E, and let $\alpha, \beta \geq 0$. Show that $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$. [6]

OR

(d) Let f be defined and bounded on a measurable set E with mE finite. If f is measurable, then show that $\inf_{f\leq\psi} \int_E \psi(x)dx = \sup_{f\geq\varphi} \int_E \varphi(x)dx$, where ψ and φ are simple functions. [6]

Q.5

(e) If $\{f_n\}$ is a sequence of nonnegative measurable functions and $f_n(x) \to f(x)$ almost everywhere on a set E, then show that $\int_E f \leq \liminf_n \int_E f_n$. Hence prove Monotone Convergence Theorem. [6]

(f) Prove that the Lebesgue integral of a nonnegative measurable function generates a measure. [6]

OR

(f) When do we say that a sequence $\{f_n\}$ converges in measure? Let $\{f_n\}$ be a sequence of measurable functions that converges in measure to f. Show that there is a subsequence $\{f_{n_k}\}$ that converges to f almost everywhere. [6]

Q.6

(g) If f is integrable on $[a,b]$ and $\int_a^x f(t)dt = 0$ for all $x \in [a,b]$, then show that $f = 0$ a.e. in $[a,b]$. State the results you use. [6]

(h) Show that every absolutely continuous function on $[a,b]$ is of bounded variation. Is the converse true? Justify. [6]

OR

(h) Let f and g be real valued functions on $[a,b]$, and let $\alpha \in \mathbb{R}$. Show that $T_a^b(f+g) \leq T_a^b(f) + T_a^b(g)$ and $T_a^b(\alpha f) = |\alpha| T_a^b(f)$. [6]

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[SEAT No. ________]

No. of printed pages: 2

[564A-22] SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - II Examination

Monday, 06th November, 2017

PS02CMTH02, Algebra-I

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: Figures to the right indicate full marks of the respective questions. Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) _____ is not a Euclidean ring.
(i) $\mathbb{Z}[x]$ (ii) $\mathbb{Z}[i]$ (iii) $\mathbb{Z}$ (iv) $\mathbb{Q}$

(b) _____ contains a zero divisor.
(i) $\mathbb{Z}$ (ii) $3\mathbb{Z}$ (iii) $\mathbb{Z}\times\mathbb{Z}$ (iv) $\mathbb{Q}$

(c) $x^3+1=(x+1)^3$ in _____.
(i) $\mathbb{Z}_3[x]$ (ii) $\mathbb{Z}_2[x]$ (iii) $\mathbb{Z}[x]$ (iv) none of the other three

(d) _____ is irreducible in $\mathbb{Z}_2[x]$.
(i) x^2+1 (ii) x^3+1 (iii) x^2-x (iv) x^2-x+1

(e) The field $\mathbb{R}[x]/\langle x^2+1\rangle$ is isomorphic to _____.
(i) $\mathbb{Q}$ (ii) $\mathbb{Q}(\sqrt{2},\sqrt{3})$ (iii) $\mathbb{C}$ (iv) $\mathbb{R}$

(f) $[\mathbb{C}:\mathbb{Q}]=$_____.
(i) 1 (ii) 2 (iii) 3 (iv) ∞

(g) _____ is not a normal extension of $\mathbb{Q}$.
(i) $\mathbb{Q}$ (ii) $\mathbb{Q}(2^{\frac{1}{3}})$ (iii) $\mathbb{Q}(i)$ (iv) $\mathbb{Q}(\sqrt{3})$

(h) _____ is a symmetric rational function in x_1,x_2,x_3.
(i) $(x_1x_2x_3)^2$ (ii) x_1+x_2 (iii) $x_1x_2+x_2x_3$ (iv) x_1x_2

Q-2 Attempt *Any Seven* of the following: [14]

(a) Mention all units in $\mathbb{Z}[i]$.

(b) Show that $\phi:\mathbb{C}\to\mathbb{C}$ defined by $\phi(x+iy)=x-iy$, $(x+iy\in\mathbb{C})$, is a homomorphism.

(c) Show that a cubic polynomial in $\mathbb{Q}[x]$ has at least one real root.

(d) Find all cubic irreducible polynomials in $\mathbb{Z}_2[x]$.

(e) Show that x^4+x+1 is irreducible in $\mathbb{Z}_2[x]$.

(f) Find a polynomial in $\mathbb{Q}[x]$ whose one root is $\sqrt{4}+\sqrt{3}$

(g) Is $\{\frac{p(\pi)}{q(\pi)}:p(x),q(x)\in\mathbb{Q}[x]\}$ a field? Justify your answer.

(h) Find $G(\mathbb{Q}(i),\mathbb{Q})$.

(i) Define the term: *solvable group* and give one example of the same.

Q-3 (j) If $\mathcal{I}$ is an ideal of Euclidean ring $\mathcal{R}$, then show that $\mathcal{I} = \langle a_0 \rangle = \{a_0 x : x \in \mathcal{R}\}$ for some $a_0 \in \mathcal{R}$. [6]

(k) Show that any two nonzero elements a and b in an Euclidean ring $\mathcal{R}$ have greatest common divisor and $(a, b) = \lambda a + \mu b$ for some $\lambda, \mu \in \mathcal{R}$. [6]

OR

(k) State and prove unique factorization theorem. [6]

Q-4 (l) State and prove Gauss's lemma. [6]

(m) State and prove the Eisenstein criterion. [6]

OR

(m) Show that $f(x) = \frac{3}{7}x^4 - \frac{2}{7}x^2 + \frac{9}{35}x + \frac{3}{5}$ is irreducible over $\mathbb{Q}$. [6]

Q-5 (n) If L is a finite extension of K and K is a finite extension of F, then show L is a finite extension of F. [6]

(o) Let K be an extension of F and $a \in K$. If a is algebraic over F, then show that $F(a)$ is a finite extension of F. [6]

OR

(o) Let K be an extension of F. If $a, b \in K$ are algebraic over F then show that ab is algebraic over F. [6]

Q-6 (p) Consider $F[x_1, x_2, \ldots, x_n]$ and the elementary symmetric functions $a_1, a_2, \ldots, a_n$ [07] [6] in the variables $x_1, x_2, ..., x_n$. Show that the splitting field of the polynomial

$$p(t) = t^n - a_1 t^{n-1} + a_2 t^{n-2} - \cdots + (-1)^{n-1} a_{n-1} t + (-1)^n a_n \in F(a_1, a_2, ..., a_n)[t]$$

is $F(x_1, x_2, ..., x_n)$.

(q) Let $n > 1$ be an integer, F be a field which contains all n^{th} roots of unity and $a \in F \setminus \{0\}$. Let K be the splitting field of $x^n - a \in F[x]$, then show that $K = F(u)$, where $u \in K$ is any root of $x^n - a$. [6]

OR

(q) In usual notations prove that $[F(x_1, x_2, \ldots, x_n) : S] = n!$. [6]

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(129 & A-61)

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No of printed pages: 2

Sardar Patel University

Mathematics

M.Sc. Semester II

Wednesday, 08 November 2017

2.00 p.m. to 5.00 p.m.

PS02CMTH03 – Differential Geometry

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) A Cartesian representation of the curve $\overline{\gamma}(t) = (\cos^2 t, \sin^2 t)$, $t \in \mathbb{R}$, is

(a) $x + y = 1$ (b) $x + y = 0$ (c) $x - y = 1$ (d) none of these

(2) Which of the following curves is not a closed curve?

(a) $x^2 + y^2 = 1$ (b) $x^2 + y^2 = 4$ (c) $x^2 - y^2 = 9$ (d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

(3) Which of the following surfaces cannot be covered by a single patch?

(a) cylinder (b) torus (c) cone (d) plane

(4) The equation of the tangent plane to $x^2 + y^2 = 1$ at $(0, 1, 0)$ is

(a) $x = 0$ (b) $x^2 = 1$ (c) $x = -1$ (d) none of these

(5) The Gauss map is an identity map on

(a) sphere (b) hyperboloid (c) paraboloid (d) plane

(6) The Gaussian curvature on a plane is

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) 0

(7) Which of the following maps preserve Gaussian curvature?

(a) local diffeomorphism (c) diffeomorphism
(b) conformal (d) None of these

(8) Which of the following is not a geodesic on the unit sphere?

(a) $\overline{\gamma}(t) = (\cos t, 0, \sin t)$ (c) $\overline{\gamma}(t) = (-\cos t, 0, -\sin t)$
(b) $\overline{\gamma}(t) = (-\cos t, 0, \sin t)$ (d) $\overline{\gamma}(t) = (t, 0, t)$

Q.2 Attempt any ***Seven***. [14]

(a) Is the curve $\overline{\gamma}(t) = (t, t^2, t^2)$, $t \in \mathbb{R}$, planar? Why?

(b) Compute the torsion of the curve $\gamma(t) = (t, t^2)$.

(c) State Isoperimetric inequality.

(d) Define tangent vector and tangent space at a point on a smooth surface.

(1)

(P.T.O.)

(e) If $I : \mathcal{S} \to \mathcal{S}$ is the identity map, then show that D_pI is the identity map from $T_p\mathcal{S}$ to $T_p\mathcal{S}$ for all $p \in \mathcal{S}$.

(f) Compute second fundamental form of $\sigma(u,v) = (u, v, 2u)$.

(g) Define tangent vector field along a curve on a surface.

(h) Show that every geodesic on a surface has a constant speed.

(i) State Bonnet's Theorem.

Q.3

(a) Let $\overline{\gamma} : (a,b) \to \mathbb{R}^3$ be a regular curve with nowhere vanishing curvature. If both curvature and torsion of $\overline{\gamma}$ are constant, then show that $\overline{\gamma}$ is part of a circular helix. State the result you use. [6]

(b) Define reparametrization of a parametrized curve $\overline{\gamma} : (a,b) \to \mathbb{R}^n$. Show that a curve $\overline{\gamma} : (a,b) \to \mathbb{R}^n$ is regular if and only if it has unit speed reparametrization. [6]

OR

(b) Define vertex of a regular plane curve. Let $a, b > 0$. Determine all vertices of the curve $\overline{\gamma}(t) = (a\cos t, b\sin t)$, $t \in \mathbb{R}$. [6]

Q.4

(c) Define smooth surface. Show that the set $\{(x,y,z) \in \mathbb{R}^3 : z = 2x^2 - 3y^2\}$ is a smooth surface. [6]

(d) Show that a local diffeomorphism $f : \mathcal{S}_1 \to \mathcal{S}_2$ is a local isometry if and only if, for any surface patch σ of $\mathcal{S}_1$, the patches σ and $f \circ \sigma$ of $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively, have the same first fundamental form. [6]

OR

(d) (i) Let $f : \mathcal{S}_1 \to \mathcal{S}_2$ be a local diffeomorphism, and let $\overline{\gamma}$ be a regular curve in $\mathcal{S}_1$. Show that $f \circ \gamma$ is a regular curve in $\mathcal{S}_2$. [3]

(ii) Compute the surface area of $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 9,\ 0 < z < 9\}$. [3]

Q.5

(e) Let $\overline{\gamma}$ be a unit-speed curve, and let $\sigma(u,v) = \overline{\gamma}(u) + v\overline{n}(u)$, where $\overline{n}$ is the unit normal of $\overline{\gamma}$. Show that $\overline{\gamma}$ is planar if and only if the Gaussian curvature $\mathcal{K}$ of σ is zero. [6]

(f) Compute the principal curvatures and the principal vectors of $\sigma(u,v) = (u, v, uv)$ at $(0,0,0)$. [6]

OR

(f) Define umbilic point on a surface. Let $\sigma : U \to \mathbb{R}^3$ be a connected surface patch such that every point is an umbilic. Show that $\sigma(U)$ is a subset of a plane or a sphere. [6]

Q.6

(g) Define *geodesic* on a surface. Let $\overline{\gamma}(t) = \sigma(u(t), v(t))$ be a curve on a regular surface patch σ of a surface $\mathcal{S}$. Show that $\overline{\gamma}$ is a geodesic if and only if $\frac{d}{dt}(E\dot{u}+F\dot{v}) = \frac{1}{2}(E_u\dot{u}^2+2F_u\dot{u}\dot{v}+G_u\dot{v}^2)$ and $\frac{d}{dt}(F\dot{u}+G\dot{v}) = \frac{1}{2}(E_v\dot{u}^2+2F_v\dot{u}\dot{v}+G_v\dot{v}^2)$. [6]

(h) Define Christoffel's symbols of second kind. If σ is a regular patch, then show that $\sigma_{uu}\sigma_u = \frac{1}{2}E_u$, $\sigma_{uu}\sigma_v = F_u - \frac{1}{2}E_v$, $\sigma_{uv}\sigma_u = \frac{1}{2}F_v$, $\sigma_{uv}\sigma_v = \frac{1}{2}G_u$, $\sigma_{vv}\sigma_u = F_v - \frac{1}{2}G_u$ and $\sigma_{vv}\sigma_v = \frac{1}{2}G_v$. [6]

OR

(h) (i) Compute the Christoffel's symbols of second kind for the surface $\sigma(u,v) = (u, v, u-v)$. [3]

(ii) Show that the sum of interior angles of a triangle on sphere is strictly greater than π. [3]

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(129 & A-53)

SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. Mathematics, Semester - II

Friday, 10^{th} November, 2017

PS02CMTH04, Functional Analysis - I

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: Figures to the right indicate marks of the respective question. Assume standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions: [8]

1. Let x, y be elements of an inner product space X over $\mathbb{C}$. Then $\langle x+y, x+y\rangle - \langle x-y, x-y\rangle + i\langle x+iy, x+iy\rangle - i\langle x-iy, x-iy\rangle =$ _____.
   (a) $\frac{1}{4}\langle x, y\rangle$ (b) $4\langle x, y\rangle$ (c) $\langle x, y\rangle$ (d) 0
2. If $E = \{x, y\}$ is subset of an inner product space with $\|x\| = \|y\| = 1$ and $\langle x, y\rangle = 0$, then the diameter of E is _____.
   (a) 0 (b) 1 (c) 2 (d) $\sqrt{2}$
3. If E is a convex subset of a Hilbert space H and $x \in H$, then the number of best approximation from E to x is _____.
   (a) at most 1 (b) 0 (c) 1 (d) infinite
4. If $\{u_n\}$ is an orthonormal basis of an infinite-dimensional Hilbert space, then _____.
   (a) $\|u_n\| \to 0$ (b) $u_n \to 0$ (c) $u_n \to 0$ weakly (d) $\{u_n\}$ is Cauchy
5. Let H be a Hilbert space and $T \in BL(H)$ be one-one. Then _____.
   (a) $R(T^*) = H$ (b) $\overline{R(T^*)} = H$ (c) $\ker(T^*) = H$ (d) $\ker(T^*) = \{0\}$
6. Let H be a Hilbert space, $S, T \in BL(H)$ be self-adjoint. Then _____ is self-adjoint.
   (a) ST (b) ST^2 (c) S^2T (d) $S - T$
7. Let H be an infinite-dimensional Hilbert space over $\mathbb{C}$ and $T \in BL(H)$. If $\lambda \in \mathbb{C}$ be such that $T - \lambda I$ is not invertible, then _____.
   (a) $\lambda \in \sigma_a(T)$ (b) $\lambda \in \sigma_e(T)$ (c) $\lambda \in \sigma(T)$ (d) none of these
8. Let H be a finite dimensional Hilbert space and $T \in BL(H)$. Then _____.
   (a) T is self-adjoint (b) T is normal (c) T is unitary (d) T is compact

Q-2 Attempt *Any Seven* of the following: [14]

(a) State and prove Parallelogram law for an inner product space.

(b) Define inner product.

(c) Compute the Gram matrix of $x_1 = (2, -1, -1)$, $x_2 = (0, 1, -1)$ and $x_3 = (1, 1, 1)$.

(d) Let X be an inner product space and $f : X \to \mathbb{C}$ be defined by $f(x) = \langle x, y\rangle$, $x \in X$. Show that f is linear.

(e) Let H be a Hilbert space and $T \in BL(H)$ be bounded below. Show that T is one-one.

(f) Let H be a Hilbert space and $T \in BL(H)$. Show that $\ker(T) = \ker(T^*T)$.

(g) Let H be a Hilbert space and $T \in BL(H)$. Show that T is isometry if and only if $T^*T = I$.

(P.T.O.)

(h) Let H be a Hilbert space over K, where $K = \mathbb{R}$ or $\mathbb{C}$. Let $T \in BL(H)$ and $\lambda \in K$. Show that $\lambda \in \sigma(T)$ if and only if $\bar{\lambda} \in \sigma(T^*)$.

(i) If $\lambda, \mu \in \sigma_e(T)$, $\lambda \neq \mu$ and $x \neq 0, y \neq 0$ are such that $Tx = \lambda x$ and $Ty = \mu y$, then show that $\langle x, y \rangle = 0$.

Q-3 (a) Let X be an inner product space. Show that $|\langle x, y\rangle|^2 \leq \langle x, x\rangle\langle y, y\rangle$, for all $x, y \in X$ and the equality holds if and only if x and y are linearly dependent. [6]

(b) Let X be an inner product space and E be an orthonormal subset of X. Show that for each $x \in X$, the set $E_x = \{u \in E : \langle x, u\rangle \neq 0\}$ is countable. [6]

OR

(b) Show that $\left(\ell^p, \|\cdot\|_p\right)$ is an inner product space if and only if $p = 2$. [6]

Q-4 (a) State and prove Riesz-representation theorem. [6]

(b) Let E be a non-empty closed, convex subset of a Hilbert space H and $x \in H$. Then show that there is a unique best approximation from E to x. [6]

OR

(b) State and prove Projection theorem. [6]

Q-5 (a) Let H be a Hilbert space and $T \in BL(H)$. Show that there exists a unique $S \in BL(H)$ such that $\langle Tx, y\rangle = \langle x, Sy\rangle$ for all $x, y \in H$. [6]

(b) Let H be a Hilbert space and $T \in BL(H)$. If T^* is bounded below then show that T is onto. [6]

OR

(b) Let H be a Hilbert space and $S, T \in BL(H)$ be normal. If S commutes with T^* and T commutes with S^* then show that $S + T$ and ST are normal. [6]

Q-6 (a) Let H be a Hilbert space, $T \in BL(H)$. Show that $\sigma(T) = \sigma_a(T) \cup \{\bar{\mu} \mid \mu \in \sigma_e(T^*)\}$. [6]

(b) Let H be a Hilbert space and $T \in BL(H)$ be compact. Show that T^* is compact. [6]

OR

(b) Let H be a Hilbert space and $T \in BL(H)$ be compact. Let $\lambda \in \sigma_a(T)$ and $\lambda \neq 0$. Then show that $\lambda \in \sigma_e(T)$ and $\ker(T - \lambda I)$ is finite-dimensional. [6]

(119) SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester II) Examination

Date: 13 – 11 – 2017, Monday **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS

Paper No. PS02CMTH05 – (Methods of Partial Differential Equations) Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The equation $(D' + 3)^2(2D' + D^2)z = 0$ has (order, degree) =
(a) (3, 1) (b) (3, 2) (c) (4, 1) (d) (4, 2)

(2) The equation $p^2 + q^2 = 0$ is same as $F(D, D')z = 0$ where $F(D, D')$ is
(a) $(D + D')^2$ (b) $(D^2 + D'^2)$ (c) $D^2D' + D'^2D$ (d) none of these

(3) Which of the following equation is reducible?
(a) $2D^2 + D'$ (b) D^2D' (c) $2 + 4DD'$ (d) $3D' - D + D'^2$

(4) The equation $(9xD^2 - 3yDD' + 4yD)z = 0$ is parabolic on
(a) (0, 0) (b) X-axis only (c) Y-axis only (d) empty set

(5) Which of the following equation can be solved by general method?
(a) $x^2r - xys = 0$ (b) $x^2r - y^2s = 0$ (c) $r + t = 0$ (d) $xr + yt = 0$

(6) The equation $r + s - t = xy$ cannot be solved by
(a) General method (b) Monge's method
(c) Polynomial method (d) changing $u = \log x$, $v = \log y$

(7) The solution of Dirichlet BVP (if exists) is
(a) 0 (b) unique (c) not unique (d) none of these

(8) The Poisson Integral formula can be obtained from
(a) Dirichlet BVP (c) Harnack's Theorem
(b) Neumann BVP (d) Green's Identity

2. Attempt ANY SEVEN: [14]

(a) Define complementary function and particular integral of the equation.

(b) Eliminate functions f and g and obtain a pde: $u = f(x - 2iy) + g(x + 2iy)$.

(c) Write the equation into $\partial/\partial x$, $\partial/\partial y$ form: $D(2D'^2 + 3D'D + 2D)z = 0$.

(d) Find the complementary function of the equation: $(4D^2 - D')z = x - 2y$.

(e) Find D^2z, if x and y in $z = z(x, y)$ is replaced by $u = \log x$ and $v = \log y$.

(f) Classify region in which equation $(2xD^2 + yD'^2 - 2yD' + xD)z = 0$ is elliptic.

(g) Write the Laplace equation in cylindrical co-ordinate system.

(h) State Green's Identity.

(i) What is boundary value problem?

(P.T.O.)

3. (a) If $(\beta D' + \gamma)^2$ is a factor of $F(D, D')$, then prove that $e^{-\frac{\gamma}{\beta}y}[\varphi_1(\beta x) + y\varphi_2(\beta x)]$ is a solution of $F(D, D')z = 0$, where φ_1 and φ_2 are arbitrary functions of a single variable ξ. [6]

(b) Find the general solution of ANY ONE of the following equations: [6]

(i) $(D + 3D')(2D - D' + 1)z = x + 2y + e^{(x-2y)}$.

(ii) $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$

4. (a) Convert the following equation into canonical form: [6]

$$y(x + y)(r - s) - xp - yq = 0, \ (y \neq 0, x \neq -y)$$

(b) Using Monge's method, solve ANY ONE of the following equations: [6]

(i) $r + s - 2t = y$ (ii) $(rt - s^2) + 4 = 0$.

5. (a) Find the general solution of equation: $(x^2D^2 - 4y^2D'^2 - 6yD')z = 0$. [6]

(b) Solve the Wave equation in Cartesian coordinates, by method of separation variable and show that solution is $\psi(x,y,z,t) = e^{\pm i(lx + my + nz + kct)}$ where l, m, n and k are constants with $l^2 + m^2 + n^2 = k^2$. [6]

OR

(b) By separating the variables, find the solution of three dimensional Diffusion equation in cylindrical coordinate system. [6]

6. (a) Solve interior Dirichlet problem for a function $\phi = \phi(r, \theta)$ for circle and show that solution is of the form $\phi(r, \theta) = \sum_{n=0}^{\infty} r^n (A_n \cos n\theta + B_n \sin n\theta)$, with A_n, B_n are constants. [6]

(b) State and prove Maximum principle. [6]

OR

(b) Define equipotential surfaces and show that the family of surfaces $x^2 + y^2 + z^2 = c^2$ can form an equipotential family of sufaces. [6]

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SEAT No.________

[90]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester II) Examination

Date: 15-11-2017 **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS02EMTH02 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The diameter of the graph K_n $(n > 1)$ is

(a) 1 (b) 2 (c) n (d) $n - 1$

(2) Let T be a spanning out-tree with root R. Then

(a) $d^-(R) > 0$ (b) $d^+(R) > 0$ (c) $d^+(R) > 0, d^-(R) > 0$ (d) none of these

(3) For $G = C_n$ with clockwise direction, rank(B) is

(a) n (b) $n - 1$ (c) 1 (d) none of these

(4) If G is a complete symmetric digraph with n vertices, then $|E(G)| =$

(a) $n(n - 1)$ (b) n (c) $\frac{n(n-1)}{2}$ (d) n^2

(5) The coefficient c_5 in chromatic polynomial of P_5 is

(a) 5 (b) 5! (c) 5^2 (d) 5^5

(6) Which of the following graphs is Hamiltonian?

(a) $K_{n,1}$ (b) $K_{n,2n}$ (c) $K_{n,n}$ (d) P_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum

(b) maximal $\Rightarrow$ perfect (d) maximum $\Rightarrow$ maximal

(8) If $G = P_{51}$, then

(a) $\alpha(G) > \beta(G)$ (b) $\alpha(G) < \beta(G)$ (c) $\alpha(G) = \beta(G)$ (d) $|\alpha(G)| = |\beta(G)|$

2. Attempt any SEVEN: [14]

(a) Find radius of $K_{n,n}$ $(n > 1)$.
(b) Prove or disprove: A symmetric digraph is an Euler digraph.
(c) Define spanning in-tree and give one example of it.
(d) Define fundamental circuit matrix in a digraph.
(e) Prove: If G is a bipartite graph, then $\chi(G) = 2$.
(f) What is Four color problem?
(g) Prove: The graph K_{2n} has a perfect matching.
(h) Prove or disprove: The graph C_4 is isomorphic to $K_{2,2}$.
(i) Prove or disprove:: For any graph G, $\alpha'(G) = \beta(G)$.

3. (a) Define the following with examples: [6]
(i) Simple Digraph (ii) in-degree & out-degree (iii) Strongly Connected Digraph.

(b) Define regular digraph and balanced digraph and discuss the relation between them. [6]

OR

(b) Obtain De Bruijn cycle for r = 3 with all detail. [6]

4. (a) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

(b) Prove: An arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Prove that for each $n \geq 1$, there is a simple digraph with $n-$vertices $v_1, v_2, \ldots, v_n$ such that $d^+(v_i) = i-1$ and $d^-(v_i) = n-i$ for each $i = 1, 2, \ldots, n$. [6]

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Find chromatic polynomial for the graph $K_{2,2}$. [6]

OR

(b) Define chromatic number $\chi(G)$ of a graph G. Give an example of a non-complete graph G with $\chi(G) = \Delta(G) + 1$. [6]

6. (a) Let G be a graph (no isolated vertex) with n vertices. Prove that $\alpha'(G) + \beta'(G) = n$. [6]

(b) Prove: A matching M in graph G is maximum if and only if G has no M-augmenting path. [6]

OR

(b) Define $\alpha(G)$, $\beta(G)$ and find it with the corresponding sets for $G = K_{n,m}$ $(n \neq m)$. [6]

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SEAT No.________

No. of printed pages: 2

[91]

SARDAR PATEL UNIVERSITY

M.Sc. Mathematics, Semester - II

Wednesday, 15^{th} November, 2017

PS02EMTH04, Mathematical Classical Mechanics

Time: 02:00 p.m. to 05:00 p.m. **Maximum marks: 70**

Note: All the questions are to be answered in answer book only. Figures to the right indicate full marks of the respective question. Assume standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions: [8]

1. Motion of a particle in space is __________ constraint.
   (a) a holonomic (b) a non-holonomic (c) a rheonomic (d) not a
2. Degrees of freedom of a double pendulum is __________.
   (a) 0 (b) 1 (c) 2 (d) 3
3. If L does not depend on q_j explicitly, then __________ is conserved.
   (a) p_j (b) L (c) H (d) total energy
4. __________ is a Brachistochrone curve.
   (a) Great circle (b) Cycloid (c) Straight line (d) Catenary
5. If all the coordinates of a system are non-cyclic, then Routhian $R =$ __________.
   (a) L (b) H (c) $-L$ (d) $-H$
6. Which one of the following is correct?
   (a) $\frac{\partial H}{\partial p_j} = -q_j$ (b) $H = h$ (c) $\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$ (d) none of these
7. Which one of the following is correct?
   (a) A symplectic matrix is invertible.
   (b) Symplectic matrices does not form a group.
   (c) Canonical transformations are not invertible.
   (d) A symplectic matrix is singular.
8. For a system of 3 degrees of freedom, the Poisson bracket $[2q_1+5q_3, 3q_2] =$ __________.
   (a) 0 (b) 10 (c) 6 (d) -6

Q-2 Attempt ***Any Seven*** of the following: [14]

(a) Define rheonomic constraint and give its example.

(b) State Lagrange's equations of motion when frictional force is present.

(c) State Hamilton's equations of motion in matrix form.

(d) State the condition for extremum of $I = \int_{x_1}^{x_2} f(y, \dot{y}, \ddot{y}, x)\, dx$.

(e) For a system of n-degrees of freedom, define phase space and canonical variables.

(f) Show that a cyclic coordinate is ignorable in Hamiltonian formalism.

(g) State the transformation equations for a generating function of type F_2.

(h) In usual notations, show that $[u, u] = 0$.

(i) Define canonical transformation and show that the identity transformation $Q_i = q_i$ and $P_i = p_i$ for all $i = 1, 2, \ldots, n$ is canonical.

Q-3 (a) Using D'Alembert's principle, derive Lagrange's equations of motion in general form. [6]

(b) Stating degrees of freedom obtain Lagrange's equations of motion for a spherical pendulum. [6]

OR

(b) Derive the expression $T = T_0 + T_1 + T_2$ of kinetic energy in usual notations. [6]

Q-4 (a) Derive the condition for extremum of the line integral $\int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$. [6]

(b) The Lagrangian of a system is given by $L = \frac{m}{2}(\dot{\theta}^2 + \sin^2\theta\, \dot{\phi}^2) + mg\cos\theta$. Find the energy function. Is it conserved? Justify your answer. [6]

OR

(b) Obtain the solution of Euler's equation for Brachistochrone problem, where the function $f = \sqrt{\dfrac{1+\dot{x}^2}{2gy}}$. [6]

Q-5 (a) State Hamilton's modified principle and hence derive Hamilton's equations of motion from it. [6]

(b) Lagrangian of a system is given by $L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r}$. Derive Routhian equations of motion for the system. [6]

OR

(b) Discuss principle of least action. [6]

Q-6 (a) Describe the method of solving a dynamical problem using Poisson bracket formalism. [6]

(b) Using symplectic condition show that the following transformation is canonical: [6]

$$Q = \log\left(\frac{\sin p}{q}\right), \quad P = q\cot p.$$

OR

(b) If u is a dynamical quantity of a system and H is Hamiltonian, then show that [6]

$$\frac{du}{dt} = [u, H] + \frac{\partial u}{\partial t}.$$

Hence deduce that if u and v do not depend on t explicitly and they are constants of motion, then $[u, v]$ is a constant of motion.

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