(A-17)

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SARDAR PATEL UNIVERSITY

M.Sc. (Semester-IV) Examination

October – 2016

Saturday, 22 October, 2016

Time: 2:00 PM to 5:00 PM

Subject: Mathematics

Course No.PS04EMTH15 (Relativity-II) Total Marks: 70.

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.
(3) Use the expressions in the Appendix if necessary.

Q-1 Choose most appropriate answer from the options given. (08)

(1) Associated Metric tensor is a _____ tensor.
(a) covariant (b) contravariant (c) inverse (d) mixed

(2) Which one of the following is true?
(a) Christoffel symbols are scalars (b) Christoffel symbols are tensors
(c) Christoffel symbols are invariants (d) Christoffel symbols are not tensors

(3) The necessary and sufficient condition for a flat space is given by
(a) $R_{hijk} = 0$ (b) $R_{ij} = 0$ (c) $R_{ij} \neq 0$ (d) $R_{hijk} \neq 0$

(4) In Schwarzschild exterior metric $r = 0$ _____
(a) is removable singularity
(b) is a regular singular point
(c) lies outside the region of validity of the solution
(d) is not a singularity

(5) Which one of the following is not a crucial test of relativity?
(a) Advance of perihelion of a planet (b) Singularity at r=2m
(c) Bending of light (d) all of above

(6) Which one of the following is correct?
(a) de-Sitter universe is flat
(b) de-Sitter universe is spherically symmetric
(c) de-Sitter universe is non-static
(d) de-Sitter universe is non-empty

(7) Which one of the following is non-static model?
(a) Einstein model (b) Friedmann model
(c) Schwarzschild interior solution (d) none of above

(8) For Robertson-Walker metric which one of the following is correct?
(a) It is a static solution. (b) It represents expanding universe.
(c) R(t) is a constant function. (d) It admits redshift.

Q-2 Answer any **Seven.** (14)

(1) Show that gradient of a scalar is a covariant vector.
(2) State the formula for Christoffel symbols of second kind. Are they symmetric?
(3) Sate the Schwarzschild interior metric.
(4) What are singularities in Schwarzschild exterior metric?
(5) State cosmological principle.
(6) Give an example of static model of the universe.
(7) State Robertson-Walker metric.
(8) What is Hubble parameter? Explain in brief.
(9) What is meant by Friedmann model?

(P.T.O.)

Q-3

(a) Show that the expression $ds^2 = g_{ij}dx^i dx^j$ remains invariant under the transformation of coordinates. (06)

(b) State the geodesic equations, show that geodesics in three dimensional Euclidean space are straight lines. (06)

OR

(b) State Einstein's field equations show that in vacuum they reduce to vanishing of Ricci tensor.

Q-4

(a) Derive the Schwarzschild exterior metric. (06)

(b) State and prove Birkhoff's theorem. (06)

OR

(b) Explain the gravitational bending of light using relativity theory.

Q-5

(a) What is meant by a static model? Show that only three types of static models are possible. (06)

(b) Define an Einstein space. Give an example of an Einstein space with details. (06)

OR

(b) State metric for Einstein universe. Obtain the expression of the volume spherical universe.

Q-6

(a) State Weyl's hypothesis and cosmological principle. (06)

(b) For the Robertson-Walker model show that $\frac{\dot{\rho}}{\rho+p} + \frac{3\dot{R}}{R}=0$. (06)

OR

(b) For the Friedman spaces, show that $R(t) = \left(\frac{3}{2}H_0 t\right)^{2/3}$.

Appendix

For the spherically symmetric static metric $ds^2 = -e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu(r)}dt^2$, non-zero independent Christoffel symbols and components of Ricci tensor are as under,

$\Gamma^1_{11} = \frac{\lambda'}{2}$, $\Gamma^1_{22} = -re^{-\lambda}$, $\Gamma^1_{33} = -re^{-\lambda}\sin^2\theta$, $\Gamma^1_{44} = e^{\nu-\lambda}\frac{\nu'}{2}$, $\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r}$, $\Gamma^2_{33} = -\sin\theta\cos\theta$, $\Gamma^3_{23} = \cot\theta$, $\Gamma^4_{14} = \frac{\nu'}{2}$

$$R_{11} = \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} - \frac{\lambda'}{r},\ R_{22} = -1 + e^{-\lambda} - \frac{1}{2}re^{-\lambda}(\lambda' - \nu'),$$

$$R_{33} = \sin^2\theta\, R_{22},\quad R_{44} = -e^{\nu-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} + \frac{\nu'}{r}\right]$$

For R-W metric the non-zero components of Ricci tensor are given by,

$$\frac{R_{11}}{g_{11}} = \frac{R_{22}}{g_{22}} = \frac{R_{33}}{g_{33}} = \frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}^2+k}{R^2}\right),\quad \frac{R_{44}}{g_{44}} = \frac{3\ddot{R}}{R}$$

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