[49 & A-18] Seat No ——

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester IV
Thursday, 20 October 2016
2.00 p.m. to 5.00 p.m.
PS04CMTH01 - Complex Analysis II

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) The value of the index number on the unbounded component is ____

(a) 0 (b) 1 (c) ∞ (d) none of these

(2) The Casoratti-Weierstrass Theorem deals with the behavior of an function near ____

(a) pole (c) essential singularity
(b) removable singularity (d) None of these

(3) The number of zeros of $z^4 + 1$ in the first quadrant is ____

(a) 1 (b) 2 (c) 3 (d) 4

(4) Let f be analytic on the open unit disc $|z| < 1$. Suppose that $|f(z)| \leq 1$ for every z and $|f(z_0)| = |z_0|$ for some nonzero z_0. Then ____

(a) $|f'(0)| = 1$ (b) $|f'(0)| \leq 1$ (c) $|f'(0)| > 1$ (d) none of these

(5) The integral of $\frac{2z+1}{z^2+6z+9}$ along the positively oriented circle $|z| = 2$ is ____

(a) πi (b) $2\pi i$ (c) $4\pi i$ (d) none of these

(6) ____ Theorem provides the example of an infinite dimensional space in which closed and bounded subsets are compact.

(a) Hadamard's Theorem (c) Schwarz's Lemma
(b) Riemann Mapping Theorem (d) Montel's Theorem

(7) The infinite product $\prod_{n=1}^{\infty}(1 + \frac{1}{n^z})$ converges if ____

(a) Re $z > 2$ (b) Re $z < 2$ (c) Im $z < 2$ (d) none of these

(8) With usual symbols $\varphi_a'(a)$ is ____

(a) $\frac{1}{1-|a|^2}$ (b) $1 - |a|^2$ (c) $\frac{1-|a|^2}{1+|a|^2}$ (d) none of these

Q.2 Attempt any *Seven*. [14]

(a) Show that $-n(\gamma; a) = n(-\gamma; a)$.

(b) State Morera's Theorem.

(c) Define a normal family.

(d) Let γ be a closed rectifiable curve and a does not belong to the trace of γ. Then find out the value of $\int_\gamma (z-a)dz$.

(e) State Cauchy Integral Formula in First Version.

(f) Show that straight lines are convex.

(g) Prove that $H(G)$ is a closed subset of $C(G, \mathbb{C})$.

(h) State Hurwitz's Theorem.

(i) Suppose an infinite product $\prod_n z_n$ is absolutely convergent and $\operatorname{Re} z_n > 0$ for all n. Then show that it is always convergent.

Q.3

(a) Prove that $n(\gamma; a)$ is a continuous function with respect to a. [6]

(b) Let G be an open connected set, $f : G \to \mathbb{C}$ be analytic. Suppose $\{z \in G : f(z) = 0\}$ has a limit point in G. Show that there is a point $a \in G$ such that $f^{(n)}(a) = 0$ for all $n \in \mathbb{N} \cup \{0\}$. [6]

OR

(b) State and prove Cauchy's Integral Formula in Second Version. [6]

Q.4

(c) State and prove Rouche's Theorem. [6]

(d) State and prove Counting Zero Principle and illustrate it by an example. [6]

OR

(d) Deduce Fundamental Theorem of algebra from a well known result. State the result used here. [6]

Q.5

(e) Define a locally bounded family. If a family $\mathscr{F}$ is normal, then show that it is locally bounded. [6]

(f) Suppose $f_n, f \in H(G)$. If $f_n \to f$ in H(G), then show that $f_n^{(k)} \to f^{(k)}$ in $H(G)$ for all $k \geq 1$. [6]

OR

(f) State and prove Schwarz's lemma. [6]

Q.6

(g) State Weierstrass Factorization Theorem and find a factorization of $\sin z$. [6]

(h) Let $\operatorname{Re} z_n > -1$. Then show that the series $\sum_n \log(1 + z_n)$ converges absolutely if and only if the series $\sum_n z_n$ converges absolutely. [6]

OR

(h) State and prove Riemann's Theorem on removable singularity. [6]

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