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SARDAR PATEL UNIVERSITY

M.Sc. Mathematics (Semester - IV) Examination
Friday, April 20, 2018
PS04EMTH30, Operations Research

Time: 02:00 p.m. to 05:00 p.m. **Maximum marks: 70**

Note: (1) All the questions are to be answered in the answer book only.
(2) Figures to the right indicate marks of the respective question.
(3) Assume standard notations wherever applicable.
(4) Use of calculators in the examination is permitted.

Q-1 Choose the most appropriate answer from the options given. (08)

1. For an LP in variables x_1, x_2 with constraints $x_1 = 2, x_2 \leq 5, x_1 + x_2 \leq 20$, and $x_1, x_2 \geq 0$, the solution space is ______.
   (a) singleton (b) infinite (c) infeasible (d) unbounded
2. The right hand side of all the constraints of an LP expressed in equation form is ______.
   (a) zero (b) positive (c) non-positive (d) non-negative
3. In simplex method, a tie for leaving variable gives ______ solution in the next iteration.
   (a) degenerate (b) non-degenerate (c) infeasible (d) optimum
4. If the primal is maximization type, then all the constraints in the dual are of ______ type.
   (a) $\leqslant$ (b) $=$ (c) $\geqslant$ (d) $<$
5. ______ method cannot be applied to solve a general transportation problem.
   (a) Least-cost (c) Northwest-corner
   (b) Hungarian (d) Vogel approximation
6. ______ is not a feasible solution for a 3×3 assignment problem.
   (a) $x_{11} = 1, x_{22} = 1, x_{33} = 1$ (c) $x_{13} = 1, x_{21} = 1, x_{32} = 1$
   (b) $x_{12} = 1, x_{23} = 1, x_{31} = 1$ (d) $x_{11} = 1, x_{23} = 1, x_{33} = 1$
7. Strictly unimodal single-variable functions can be optimized by the ______ method.
   (a) direct search (b) simplex (c) dual simplex (d) Hungarian
8. In a separable problem with three breakpoints, the corresponding non-negative weights w_1, w_2, and w_3 satisfy the condition ______.
   (a) $w_1+w_2+w_3 = 0$ (b) $w_1+w_2+w_3 < 1$ (c) $w_1+w_2+w_3 = 1$ (d) $w_1+w_2+w_3 = 3$

Q-2 Attempt *Any Seven* of the following: (14)

(a) How many basic solutions does an LP with n decision variables and m linearly independent equation constraints, where $n > m$ has? Why?

(b) State the penalty rule for artificial variables in objective function for Big M-method.

(c) When can we say that an LP has infinite number of alternative optima?

(d) Write the formula to determine the optimal values of dual variables from the given optimal primal inverse.

(e) What conditions must be satisfied before solving an LP by the dual simplex method?

(f) When a dummy destination is added to the transportation model?

(g) Explain how an assignment problem is a special case of a transportation problem.

(h) Define a separable function and give an example of it.

(i) Express the quadratic problem in matrix form: Maximize $z = 6x_1+3x_2-4x_1x_2-2x_1^2-3x_2^2$ subject to $x_1 + x_2 \leq 1$, $2x_1 + 3x_2 \leq 4$, and $x_1, x_2 \geq 0$.

(P.T.O.)

Q-3 (a) State the optimality and feasibility condition, and Gauss-Jordan row operations in the simplex method. (06)

(b) JOBCO manufactures two products on two machines. A unit of product 1 requires 2 hours on machine 1 and 1 hour on machine 2. For product 2, one unit requires 1 hour on machine 1 and 3 hours on machine 2. The profit per unit of products 1 and 2 are ₹30 and ₹20, respectively. The total daily available processing time for each machine is 8 hours. Compute the maximum profit by graphical method. Also, determine whether it is advisable to increase the capacity of machine 1 from 8 hours to 9 hours at ₹10 cost. (06)

OR

(b) Solve the LP by Big M-method: Maximize $z = 2x_1 + 2x_2 + 4x_3$ subject to $2x_1 + x_2 + x_3 \leq 2$, $3x_1 + 4x_2 + 2x_3 \geq 8$, and $x_1, x_2 \geq 0$. (06)

Q-4 (a) Check optimality of the solution with basic variables $= (x_2, x_1)$ and inverse $= \begin{pmatrix} \frac{7}{45} & -\frac{2}{45} \\ -\frac{2}{45} & \frac{7}{45} \end{pmatrix}$ for the LP: Maximize $z = 4x_1 + 14x_2$ subject to $2x_1 + 7x_2 + x_3 = 21$, $7x_1 + 2x_2 + x_4 = 21$, and $x_1, x_2, x_3, x_4 \geq 0$. (06)

(b) Describe the steps to formulate the dual problem from a given LP primal. (06)

OR

(b) Express the LP in equation form and hence write its dual: Maximize $z = x_1 + x_2$ subject to $2x_1 + x_2 = 5$, $3x_1 - x_2 = 6$, and x_1, x_2 both unrestricted. (06)

Q-5 (a) Solve the LP by dual simplex method: Minimize $z = 2x_1 + 3x_2$ subject to $2x_1 + 2x_2 \leq 3$, $x_1 + 2x_2 \geq 1$, and $x_1, x_2 \geq 0$. (06)

(b) Describe the Vogel approximation method to obtain the starting solution of a transportation problem. (06)

OR

(b) Solve the following assignment problem by Hungarian method. (06)

	I	II	III	IV
A	1	4	6	3
B	9	7	10	9
C	4	5	11	7
D	8	7	8	5

Q-6 (a) Express the following separable nonlinear problem into linear form by using appropriate breakpoints and weights. (06)
Maximize $z = x_1 + x_2^4$ subject to $3x_1 + 2x_2^2 \leq 9$, $x_2 \leq 3$, and $x_1, x_2 \geq 0$.

(b) Explain the gradient method for maximizing twice continuously differentiable function $f(\mathbf{X})$ using steepest ascent in case of unconstrained problems. (06)

OR

(b) By Dichotomous method, find the interval of uncertainty I_2 after two iterations for (06)

$$\text{Maximize } f(x) = \begin{cases} 3x, & 0 \leq x \leq 2 \\ \frac{1}{3}(-x + 20) & 2 \leq x \leq 3. \end{cases}$$

Assume initial interval $I_0 = (0, 3)$ and $\Delta = 0.1$ as the desired level of accuracy.

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