SEAT No._____ SC

[11] No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester IV) Examination

Date: 25-4-2018 **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS04EMTH29 – (Graph Theory – II)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The number of spanning trees in K_6 is
(a) 6 (b) 6^2 (c) 6^3 (d) 6^4

(2) For a star graph, Pruffer code has ____digits.
(a) only two different (b) all different (c) all same (d) none of these

(3) A shortest spanning tree in a graph can be obtained using
(a) BFS algorithm (b) Kruscal algorithm
(c) Dijkstra's algorithm (d) none of these

(4) In a network, if s is source and t is sink, then
(a) $d^+(s) > 0, d^-(t) = 0$ (b) $d^+(s) = 0 = d^-(t)$
(c) $d^+(s) > 0, d^-(t) > 0$ (d) $d^+(s) = 0, d^-(t) > 0$

(5) Let A be a matrix with spectrum $\{-2, 2, 3\}$. Then spectrum of A^2 =
(a) $\{-2,2,3\}$ (b) $\{4,9\}$ (c) $\{2,4,3,6,9\}$ (d) $\{-4,4,-6,6,9\}$

(6) Let $G = K_{4,6}$. Then the non-zero eigen values for G is
(a) 12 (b) 6 (c) 2 (d) 4

(7) The Ramsey number R(3, 3)
(a) = 3 (b) = 6 (c) = 9 (d) none of these

(8) If $E = \{1, 2, 3\}$ with $M = \{\phi, \{1\}, \{2\}, \{3\}\}$ as hereditary system, then r(E) is
(a) 1 (b) 2 (c) 3 (d) 0

2. Attempt any SEVEN: [14]

(a) Give one graceful labeling of P_6 with detail.

(b) Define contraction of a graph by an edge and give one example of it.

(c) Define flow and cut in a network.

(d) Define u-v vertex separating set and give one example of it.

(e) Let A be a matrix with spectrum $\{-2, -1, 2, 3, 1\}$. Then find det(A).

(f) If $G = C_7$, then find $\lambda_{max}(G)$.

(g) Prove: The Ramsey number $R(2, p) = p, \forall\ p \geq 2$.

(h) Show that $E = \{1, 2, 3\}$ with base $B_M = \{\{1, 2\}, \{1, 3\}\}$ is a graphic matroid.

(i) If $E = \mathbb{Z}$ with $M = \{X \subset E; |X| < 7\}$ as hereditary system, then find B_M & C_M.

[P. T.O.]

3. (a) How many trees are there with degree sequence (3,1,1,1,1,3)? Construct any one such tree. [6]

(b) Find $\tau(G)$ using Matrix-Tree theorem, for $G = K_{2,3}$. [6]

OR

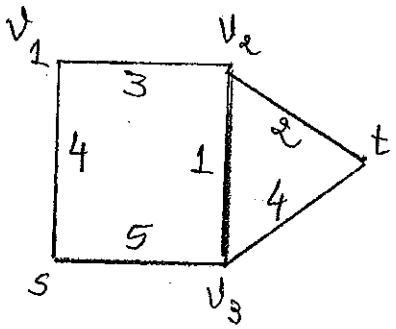
(b) Show that if a tree T with m edges has graceful labeling, then K_{2m+1} can be decomposed into (2m + 1) copies of T. [6]

4. (a) Using Dijkstra's algorithm, find a least weight path from s to t in graph (G): [6]

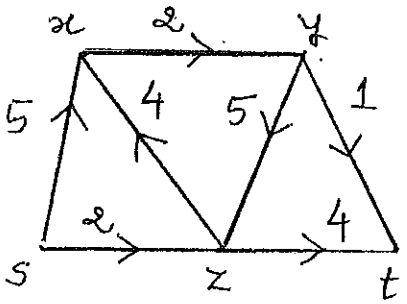
(b) Prove that in every network, there exists a flow with maximum value equals the minimum capacity of cuts. [6]

OR

(b) List all the cuts and find a minimum cut for the following network (N). [6]



(G)



(N)

5. (a) Prove: A graph G is regular and connected if and only if J is a linear combination of powers of A(G). [6]

(b) (i) Prove: If G is a bipartite graph, then non-zero eigen values of G occur in pair $(\lambda,-\lambda)$. [6]

(ii) Give an example of a non-complete graph G with $\chi(G) < 1 + \lambda_{\max}(G)$.

OR

(b) (i) Prove: For any graph G, $\lambda_{max}(G) \leq \Delta(G)$. [6]

(ii) Find $sp(K_{2,2})$.

6. (a) Prove: $R(p, q) \geq (p-1)(q-1) + 1$. [6]

(b) Prove (ANY ONE): In a hereditary system, [6]

(i) Sub modularity property (R) $\Rightarrow$ Weak elimination property (C).

(ii) Uniformity property (U) $\Rightarrow$ Base Exchange property (B).

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