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SARDAR PATEL UNIVERSITY

M.Sc. (Semester-IV) Examination

April – 2018

Tuesday, 17 April, 2018

Time: 02:00 PM to 05:00 PM

Subject: Mathematics **Course No.: PS04EMTH15 (Relativity-II)**

Total Marks: 70

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.
(3) Use the expressions in the Appendix if necessary.

Q-1 Choose most appropriate answer from the options given. (08)

(1) The type of associated metric tensor is _____ tensor.
(a) a contravariant (b) a covariant (c) a mixed (d) not a

(2) Which one of the following is true?
(a) Christoffel symbols form a tensor of (1,2) type.
(b) Christoffel symbols form a tensor of (2,1) type.
(c) Christoffel symbols are symmetric.
(d) none of the above

(3) A region is empty then _____
(a) $R_{hijk} = 0$ (b) $R_{ij} = 0$ (c) $R_{ij} = -R_{ji}$ (d) $R_{hijk} = -R_{ihjk}$

(4) Which one of the following is correct?
(a) Schwarzschild exterior solution is valid in empty region.
(b) Schwarzschild interior solution is valid in empty region.
(c) Schwarzschild exterior solution represents correct model of the universe.
(d) none of the above

(5) In Schwarzschild metric $r = 0$ _____
(a) is a removable singularity (b) always lies inside the body
(c) is a regular point (d) is an essential singularity

(6) Advance of perihelion of a planet_____.
(a) can not be explained using general relativity (b) can not be observed
(c) is a crucial test of general relativity (d) occurs due to vacuum

(7) Which one of the following is correct?
(a) de-Sitter universe is an Einstein space.
(b) Einstein universe is an Einstein space.
(c) For Einstein universe $R_{ij} = 0$.
(d) None of the above

(8) Which one of the following is incorrect?
(a) In Robertson-Walker metric R is a decreasing function
(b) In Robertson-Walker metric R is an increasing function
(c) In Robertson-Walker metric R is an oscillating function.
(d) None of the above.

Q-2 Answer any **Seven.** (14)

(1) State expressions of Christoffel symbols of both kinds.
(2) What is the maximum number of independent components of Ricci tensor in a four dimensional space?
(3) What are the boundary conditions in the Schwarzschild exterior solution?
(4) What is meant by advance of perihelion of a planet?
(5) State energy-momentum tensor for a perfect fluid. What is condition for radiation?
(6) What are possible spherically symmetric static models?

(P.T.O.)

(7) State the line element for de-Sitter universe.
(8) Define Einstein space.
(9) State Robertson-Walker metric.

Q-3

(a) State the expression of Riemann tensor and its algebraic properties. (06)
(b) Obtain geodesic equations on a sphere and hence obtain geodesics on it. (06)

OR

(b) Define covariant derivative of tensors of rank two also find $g^{ij}{}_{;k}$.

Q-4

(a) What are singularities in Schwarzschild exterior metric? Discuss Eddington-Finkelstein coordinates for removing singularity in this metric. (06)
(b) Show that Einstein's field equations are satisfied by Schwarzschild exterior metric. (06)

OR

(b) Discuss bending of light using Schwarzschild exterior metric.

Q-5

(a) Derive the metric for Einstein universe. (06)
(b) Discuss redshift in de-Sitter universe. (06)

OR

(b) State metric for Einstein universe. Obtain the expression of volume of Einstein spherical universe.

Q-6

(a) State cosmological principle and Weyl hypothesis. Discuss importance of these two. (06)
(b) State Robertson-Walker metric and in usual notations show that $\frac{R_0}{R} = 1 + z$. (06)

OR

(b) What is meant by Friedman models? Discuss various forms of these models.

Appendix

For the spherically symmetric static metric $ds^2 = -e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu(r)}dt^2$, non-zero independent Christoffel symbols and components of Ricci tensor are as under,

$$\Gamma^1_{11} = \frac{\lambda'}{2},\ \Gamma^1_{22} = -re^{-\lambda},\ \Gamma^1_{33} = -re^{-\lambda}\sin^2\theta,\ \Gamma^1_{44} = e^{\nu-\lambda}\frac{\nu'}{2},\ \Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r},\ \Gamma^2_{33} = -\sin\theta\cos\theta,\ \Gamma^3_{23} = \cot\theta,\ \Gamma^4_{14} = \frac{\nu'}{2}$$

$$R_{11} = \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} - \frac{\lambda'}{r},\ R_{22} = -1 + e^{-\lambda} - \frac{1}{2}re^{-\lambda}(\lambda' - \nu'),$$

$$R_{33} = \sin^2\theta\, R_{22},\ R_{44} = -e^{\nu-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} + \frac{\nu'}{r}\right]$$

For R-W metric the non-zero components of Ricci tensor are given by,

$$\frac{R_{11}}{g_{11}} = \frac{R_{22}}{g_{22}} = \frac{R_{33}}{g_{33}} = \frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}^2 + k}{R^2}\right),\quad \frac{R_{44}}{g_{44}} = \frac{3\ddot{R}}{R}$$