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[88]

Sardar Patel University

M.Sc. (Sem-IV), PS04EMTH13, Financial Mathematics-II;
Friday, 13^{th} April, 2018; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks; (iii) The required table of normal distribution is attached with this question paper.

Q.1 Answer the following. [8]

1. If a bank offers interest at a nominal rate of 10% compounded quarterly, what is the effective interest rate per year ?
   (A) 10.58% (B) 10.00% (C) 9.57% (D) none of these
2. The yearly cash flow sequence $20, 10$ is preferable to the sequence $0, 34$, if the yearly interest rate is
   (A) 4% (B) 8% (C) 10% (D) All of these
3. The value of European option for an asset providing discrete dividend is given by
   (A) $f(S(t_d^+), t_d^-) = f(S(t_d^+)(1 - d_y), t_d^-)$
   (B) $f(S(t_d^-), t_d^-) = f(S(t_d^-)(d_y - 1), t_d^+)$
   (C) $f(S(t_d^-), t_d^-) = f(S(t_d^-)(1 - d_y), t_d^+)$
   (D) none of these
4. The value of futures contract is
   (A) $S_0 e^{(r+r_f)T}$ (B) $S_0 e^{(r-r_{eff})T}$ (C) $S_0 e^{-(r_f-r)T}$ (D) none of these
5. A long position in $- - - - -$ option should be hedged by a long position of delta number of underlying assets.
   (A) American put (B) European put
   (C) European call (D) None of these
6. The greek letter theta of a call option is always
   (A) -1 (B) ≤ 0 (C) 0 (D) None of these
7. The standard deviation of a random variable X is given by
   (A) $E(X^2) - [E(X)]^2$ (B) $[E(X)]^2 - E(X^2)$
   (C) $E(X) - E(X^2)$ (D) none of these
8. Binomial tree can be applicable for finding value of
   (A) European call option only (B) European call and put options only
   (C) European put option only (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define effective interest rate with an example.
(b) Give a formula that approximates the number of years it would take your funds to quadruple if you received interest at a rate r compounded continuously.
(c) If you borrow \$1000 for one year with an interest rate of 8% per annum with compounded quarterly, how much do you owe at the end of the year ?
(d) Define dividend and dividend yield.
(e) Write down put-call parity and BSM differential equation for European options on an asset providing dividend yield.
(f) Define greek letter vega for option. What is vega for European call option?
(g) Write down formulas of gamma for European options. Is it non positive for both options?
(h) Define mean of a discrete random variable.
(i) Write down formula of two step binomial model.

(P.T.O.)

Q.3

(a) Jerry decides to save for her retirement starting on her 25^{th} birthday. She puts £1000 in the beginning of a year in an investment that earns interest rate 10% per annum compounded annually. She does this for 20 years and then stops adding more money for investment. She then leaves that invested money until she is 65, when she retires. Tom, Jerry's twin brother, does not save for his retirement until his 45^{th} birthday, and then he starts investing a fixed amount in the beginning of each year at 10% per annum for 20 years. How much does Tom have to invest per year to have the same amount of money as Jerry when she retires? [6]

(b) A person takes a loan for the amount L that is to be paid back over n months with equal payments of A at the end of each month. The interest rate for loan is r per month, compounded monthly. (i) In terms of L , n, and r, what is the value of A ? (ii) After payment has been made at the month j, how much additional loan principal remains ? (iii) How much of the payment during month j is for interest and how much is for principal reduction ? [6]

OR

(b) Define yield curve and present value function and derive it for $r(s) = \frac{1}{1+s}r_1 + \frac{s}{1+s}r_2$, where r_1 and r_2 are constant interest rates.

Q.4

(a) Derive the BSM formulas of an asset providing a constant dividend yield. [6]

(b) Suppose that 2 - year interest rates in United States and Australia are 6% and 8% per annum with continuously compounding, respectively, and the spot exchange rate between US dollar to Australian dollar is 1.25 Australian dollars per US dollar. Find the value of forward contract and justify it. [6]

OR

(b) Calculate the value of an eight month European put option on a currency with a strike price of 0.50. The current exchange rate is 0.52, the volatility is 12% per annum, the domestic and the foreign risk free interest rates are 4% and 8% per annum with continuously compounding, respectively.

Q.5

(a) Explain: 'a short position in a European call option should be hedged by a long position of delta number of underlying assets' with an example. [6]

(b) Explain Generalized one step binomial model. [6]

OR

(b) Find the greek letter phi for European currency call option.

Q.6

(a) Determine p, u and d in terms of σ, r and Δt using binomial model. [6]

(b) Calculate the price of a 9 months American call option on a non dividend paying stock when the current stock price is \$198, the strike price is \$200, the risk free interest rate is 8% per annum with continuously compounding and the volatility is 30% per annum. Use a binomial model with the time interval of length 3 months. [6]

OR

(b) A 2-months American put option on a stock has an exercise price of \$480. The current stock price is \$484, the risk free interest rate is 10% per annum with continuously compounding, the dividend yield on the stock is 3% per annum and the volatility is 25% per annum. Using binomial model, find the value of option when the life of the option is divided into 4 subintervals of length half month.

——X——

Table for $N(x)$ When $x \leq 0$

This table shows values of $N(x)$ for $x \leq 0$. The table should be used with interpolation. For example,

$$N(-0.1234) = N(-0.12) - 0.34[N(-0.12) - N(-0.13)]$$
$$= 0.4522 - 0.34 \times (0.4522 - 0.4483)$$
$$= 0.4509$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

C.P.T.O.)

Table for $N(x)$ When $x \geq 0$

This table shows values of $N(x)$ for $x \geq 0$. The table should be used with interpolation. For example,

$$\begin{aligned} N(0.6278) &= N(0.62) + 0.78[N(0.63) - N(0.62)] \\ &= 0.7324 + 0.78 \times (0.7357 - 0.7324) \\ &= 0.7350 \end{aligned}$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000