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[100]

Sardar Patel University

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M.Sc.(Mathematics)(Sem-IV); Examination 2018;
PS04CMTH02: Mathematical Methods II;
11-04-2018; Wednesday; Time-2.00 pm to 5.00 pm; Maximum Marks 70

Note: Notations and terminologies are standard.

Q.1 Choose correct option from given four choices. [08]

(i) Which is the third form of the Euler's Equation?

(a) $\frac{\partial f}{\partial y} - y'\frac{\partial^2 f}{\partial y \partial y'} - y''\frac{\partial^2 f}{\partial y'^2} = 0$
(b) $\frac{\partial f}{\partial y} - \frac{\partial^2 f}{\partial x \partial y'} - y'\frac{\partial^2 f}{\partial y \partial y'} - y''\frac{\partial^2 f}{\partial y'^2} = 0$
(c) $\frac{\partial f}{\partial y} - \frac{\partial^2 f}{\partial x \partial y'} - y''\frac{\partial^2 f}{\partial y'^2} = 0$
(d) none

(ii) The extremum of $I[y(x)] = \int_0^1 \sqrt{1+y'^2}dx$ passing through the points $(0,2)$ and $(2,-2)$ is

(a) $y(x) = 2x+2$ (b) $y(x) = 2x-2$ (c) $y(x) = -2x+2$ (d) -2x-2

(iii) The Laplace transform $L[\sin(2x)] =$

(a) $\frac{1}{s^2+1}$ (b) $\frac{1}{s^2+2}$ (c) $\frac{2}{s^2+2}$ (d) $\frac{2}{s^2+4}$

(iv) Which of the following is a separable kernel?

(a) $K(x,t) = \log(x^t)$
(b) $K(x,t) = x^t$
(c) $K(x,t) = \sin(xt)$
(d) none

(v) For how many λ's, the integral equation $y(x) = \lambda \int_0^1 e^{x-t}y(t)dt$ has non-trivial solution?

(a) zero (b) one (c) two (d) three

(vi) For how many λ's, the integral equation $y(x) = x + \lambda \int_0^1 \sin(x-t)y(t)dt$ has solution?

(a) finitely many
(b) countably many
(c) uncountably many
(d) none

(vii) Which of the following is a Sturm-Liouville equation?

(a) $[r(x)y]' + [\lambda q(x) + p(x)]y = 0$
(b) $[r(x)y]' + [\lambda q(x) + p(x)]y' = 0$
(c) $[r(x)y'] + [\lambda q(x) + p(x)]y = 0$
(d) $[r(x)y']' + [\lambda q(x) + p(x)]y = 0$

(viii) Which of the following is Bessel's differential equation?

(a) $x^2y'' + xy' + (x^2-1)y = 0$
(b) $x^2y'' + (x^2-1)y = 0$
(c) $y'' + y' + (x^2-1)y = 0$
(d) $x^2y'' + xy' + y = 0$

Q.2 Attempt any **seven** [14]

(i) Find the extremum of $I[y(x)] = \int_a^b (1+x^2y')y'dx$.
(ii) Prove that the extremum of $I[y(x)] = \int_a^b f(y')dx$ is a straight line.
(iii) State the equations of cycloids.
(iv) State the Leibnitz's Rule for the continuously differentiable functions $\varphi(x)$ and $\psi(x)$.
(v) convert $y(x) = 1 - \int_0^x ty(t)dt$ into a differential equation.
(vi) State the Fredholm's Alternative Theorem for a compact operator F.
(vii) Prove that $L^\infty[0,1] \subset L^1[0,1]$.
(viii) State the Hermite differential equation and convert it to the Sturm-Liouville equation.
(ix) State the Laguerre differential equation and convert it to the Sturm-Liouville equation.

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Q.3

(a) Prove that if the functional $I[y(x)] = \int_{x_1}^{x_2} f(x, y, y')dx$ has the extremum value, then the integrand f satisfies the Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}(\frac{\partial f}{\partial y'}) = 0$. [6]

(b) Find the extremal of functional $\int_{-a}^{a}(\lambda y + \frac{1}{2}\mu y''^2)dx$ which satisfies the boundary conditions $y(-a) = 0$, $y'(-a) = 0$, $y(a) = 0$, $y'(a) = 0$. [6]

OR

(b) Find geodesics on the sphere of radius a. [6]

Q.4

(a) Prove that [6]

$$\int_a^x \int_a^{x_n} \cdots \int_a^{x_2} f(x_1)dx_1 \cdots dx_n = \frac{1}{(n-1)!} \int_a^x (x-t)^{n-1} f(t)dt.$$

(b) Convert the differential equation $y'' + xy' + y = 0$; $y(0) = 1$, $y(1) = 0$ into a corresponding integral equation. [6]

OR

(b) Solve the integral equation $y(x) = x^2 + \int_0^x \sinh(x-t)y(t)dt$. [6]

Q.5

(a) Let $1 < p \leq \infty$, $1 \leq q < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$, $X = L^p([a, b])$, $Y = L^q([a, b])$, and the kernel $K(\cdot, \cdot) \in L^q([a, b] \times [a, b])$ Define $F : X \to Y$ as

$$F(x)(s) = \int_a^b K(s, t)x(t)dm(t) \quad (x \in X; s \in [a, b]).$$

Prove that $F \in CL(X, Y)$. [6]

(b) Find eigenvalues and eigenfunctions of the integral equation $y(x) = \lambda \int_0^1 (2xt - 4x^2)y(t)dt$. [6]

OR

(b) Solve the integral equation $y(x) = x + \lambda \int_0^1 (1 + x + t)y(t)dt$ [6]

Q.6

(a) Find eigenvalues and corresponding eigenfunctions of the differential equation $y'' + \lambda y = 0$ on the interval $[0, 1]$ with the boundary conditions $y'(0) = 0$ and $y'(1) = 0$. [6]

(b) Convert $xy'' + (1 + 2\lambda)y' + xy = 0$ into Bessel differential equation by substitute $x^\lambda y = z$, where $\lambda \in \mathbb{R}$. [6]

OR

(b) Convert $x^3y'' + (5x^2 - x^3)y' + 4xy = 0$ with substitute $z = x^2y$ into Laguerre differential equation. [6]

THE END