SEAT No.______

[117/A-44]

No of printed pages: **2**

Sardar Patel University
Mathematics
M.Sc. Semester IV
Monday, 10 April 2017
2.00 p.m. to 5.00 p.m.
PS04CMTH01 - Complex Analysis II

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) Which of the following is not an open map?

(a) $\sin z$ (b) e^z (c) $\overline{z}$ (d) $e^{|z|}$

(2) Let $\gamma(t) = 2e^{it}$, $0 \le t \le 4\pi$. Then $\frac{1}{2\pi i}\int_\gamma \frac{dz}{3z+1} =$ _____

(a) 2 (b) 3 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

(3) Let $N \in \mathbb{N}$, and let $\overline{\gamma}(t) = (N+\frac{1}{2})\pi e^{it}$, $0 \le t \le 2\pi$. Then the value of $\frac{1}{2\pi i}\int_\gamma \cot z\, dz$ is

(a) N (b) $2N$ (c) $2N+1$ (d) $2N+2$

(4) Which of the following is not a convex function on $[0,1]$?

(a) $\sqrt{x}$ (b) x (c) x^2 (d) x^3

(5) Let $\mathbb{H}+$ and $\mathbb{H}_-$ be open right half plane and open left half plane respectively. Which of the following are not conformally equivalent?

(a) $\mathbb{D}$ and $\mathbb{C}$ (b) $\mathbb{D}$ and $\mathbb{H}_+$ (c) $\mathbb{D}$ and $\mathbb{H}_-$ (d) $\mathbb{H}_+$ and $\mathbb{H}_-$

(6) Let $f(z) = \frac{1-\cos z}{z^2}$. Then 0 is _____ of f.

(a) a removable singularity
(b) a pole of order 1
(c) a pole of order 2
(d) an essential singularity

(7) The value of $\prod_{n=2}^{\infty}(1-\frac{1}{n^2})$ is _____.

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(8) Suppose that f is a nonconstant entire function and it has infinitely many zeros. Let (a_n) be the sequence of zeros of f. Which of the following is true?

(a) $\lim_n |a_n| = 0$
(b) $\lim_n |a_n| = 1$
(c) $\lim_n |a_n|$ may not exist
(d) none of these

Q.2 Attempt any *Seven*. [14]

(a) If $n \in \mathbb{N}$, then evaluate $\int_\gamma (\frac{z}{z-1})^n dz$, where $\gamma(t) = 1 + e^{it}$, $0 \le t \le 2\pi$.

(b) Define a *rectifiable curve* in $\mathbb{C}$. If $\gamma : [a,b] \to \mathbb{C}$ is a rectifiable curve and if f is continuous on $\{\gamma\}$, then define $\int_\gamma f$.

(c) Let G be an open connected set. If $f, g : G \to \mathbb{C}$ are analytic and $fg = 0$, then show that $f = 0$ or $g = 0$.

(d) If $f : [a,b] \to \mathbb{R}$ is convex, then prove that $\frac{f(t)-f(s)}{t-s} \le \frac{f(u)-f(t)}{u-t}$ for all $a \le s < t < u \le b$.

(e) State Hadamard's Theorem.

(f) Show that $H(G)$ is a closed subset of $C(G,\mathbb{C})$.

(g) Let $\mathscr{F} \subset C(G,\Omega)$. If the closure of $\mathscr{F}$ is compact, then show that $\mathscr{F}$ is normal.

(h) Let (z_n) be a sequence of complex numbers such that $\operatorname{Re} z_n > -1$ for all n. If the series $\sum_n z_n$ converges absolutely, then show that $\sum_n \log(1+z_n)$ converges absolutely.

(i) Show that every meromorphic function is a quotient of two entire functions.

1

Q.3

(a) Let γ be a closed rectifiable curve in $\mathbb{C}$, and let $G = \mathbb{C} - \{\gamma\}$. Let $f : G \to \mathbb{Z}$ be $f(a) = n(\gamma; a)$. Show that f is constant on each component of G. Also, show that $f(a) = 0$ for all a belonging to the unbounded component of G. [6]

(b) Suppose that γ is a rectifiable curve and φ is a continuous function on $\{\gamma\}$. For each $m \in \mathbb{N}$, let $F_m(z) = \int_\gamma \varphi(w)(z-w)^{-m}dw$, $z \notin \{\gamma\}$. Show that each F_m is analytic in $\mathbb{C} - \{\gamma\}$ and that $F'_m(z) = mF_{m+1}(z)$ for every $z \in \mathbb{C} - \{\gamma\}$. [6]

OR

(b) If G be a region and if $f : G \to \mathbb{C}$ is a continuous function such that $\int_T f(z)dz = 0$ for every triangular path T in G, then show that f is analytic in G. State Cauchy's Theorem in First Version. [6]

Q.4

(c) Let G be an open connected subset of $\mathbb{C}$. Suppose that $f : G \to \mathbb{C}$ is analytic and f is not identically zero. If $a \in G$ and $f(a) = 0$, then show that there is $n \in \mathbb{N}$ and an analytic function $g : G \to \mathbb{C}$ with $g(a) \neq 0$ such that $f(z) = (z-a)^n g(z)$ for all $z \in G$. [6]

(d) State Rouche's Theorem. Deduce Fundamental Theorem of Algebra from it. Also, find the number of roots of $e^z - az^{2017} = 0$ inside $|z| = 1$, where $a \in \mathbb{C}$ and $|a| > e$. [6]

OR

(d) Suppose that f is analytic on the open unit disc $\mathbb{D}$ and $|f(z)| \leq 1$ for all $z \in \mathbb{D}$. If $a \in \mathbb{D}$ and $f(a) = \alpha$, then show $|f'(a)| \leq \frac{1-|\alpha|^2}{1-|a|^2}$. Also, show that if $|f'(a)| = \frac{1-|\alpha|^2}{1-|a|^2}$, then there is $c \in \mathbb{C}$ with $|c| = 1$ such that $f(z) = \varphi_{-\alpha}(c\varphi_a(z))$ for all $z \in \mathbb{D}$. State the result you use. [6]

Q.5

(e) Let G be an open subset of $\mathbb{C}$. Show that there is an increasing sequence (K_n) of compact subsets of G such that $\bigcup_n K_n = G$ and $K_n \subset \operatorname{int}(K_{n+1})$ for all n. Also show that every compact subset of G is contained in some K_n. [6]

(f) Suppose that G is a region and a sequence (f_n) in $H(G)$ converges to f. If $f \neq 0$, $\overline{B(a;R)} \subset G$ and $f(z) \neq 0$ for $|z-a| = R$, then show that there is an integer N such that f and f_n have the same number of zeros in $B(a;R)$ for all $n \geq N$. Hence prove that if (f_n) is a sequence in $H(G)$ converging to f in $H(G)$ and each f_n never vanishes on G, then either $f = 0$ or f never vanishes. [6]

OR

(f) When is $a \in \mathbb{C}$ called an *essential singularity* of a function f? Prove that an isolated singularity a of f is an essential singularity if and only if for any $\delta > 0$ the set $\{f(z) : 0 < |z-a| < \delta\}$ is dense in $\mathbb{C}$. [6]

Q.6

(g) State *Weierstrass Factorization Theorem.* Hence find a canonical factorization of $\sin \pi z$. [6]

(h) Let f be analytic function on a region containing $\overline{B(0;r)}$. Suppose that $a_1, a_2, \ldots, a_n$ are the zeros of f in $B(0;r)$ repeated according to their multiplicities. If $f(0) \neq 0$, then show that [6]

$$\log|f(0)| = -\sum_{k=1}^{n} \log\left(\frac{r}{|a_k|}\right) + \frac{1}{2\pi}\int_0^{2\pi} \log|f(re^{i\theta})|d\theta.$$

OR

(h) Let f be analytic on $|z| < R$. If $a = re^{i\theta}$ and $|a| < R$, then show that [6]

$$\frac{1}{2\pi}\int_0^{2\pi} |f(re^{i\theta})|^2 d\theta = \sum_{n=0}^{\infty} \left|\frac{f^{(n)}(0)}{n!}\right|^2 r^{2n}.$$

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[254 A-17] No. of Printed Pages : 2

Sardar Patel University

M. Sc. *(Fourth Semester)* Examination

Saturday, April 22, 2017

Course No. PS04ECMTH28 : *C Programming and Mathematical Algorithms II*

Time: 02.00 p.m. to 04.00 p.m. *Maximum marks: 35*

Note: Figures to the right indicates marks.

1. Choose appropriate answer to the question from the given options. [5]
   i) float r, *s; is a declaration. Which of the following is valid?
      (a) r = s; (b) s = r; (c) &r = s; (d) s = &r
   ii) Which of the following *is true*?
      (a) One **can call a** function within its definition.
      (b) One **can define** another function within the function main().
      (c) One **cannot call** a function within another function definition.
      (d) One **can call** a function without its definition / prototype.
   iii) Which of the following is valid first line of a function?
      (a) float g(float &x) (b) float g(float *x)
      (c) float g(float x+y) (d) float g(float x * y)
   iv) Suppose *double *x, y*; is a declaration and &y is 7248. If x = &y + 3;, then the content of x is _____ .
      (a) 7272 (b) 7251 (c) 7260 (d) none of these
   v) *int x = 2; void f(int a) {x = x + a} void main(){f(2); f(7);}* what is the content of x after the execution of the program?
      (a) 4 (b) 7 (c) 11 (d) 2

2. Answer any THREE of the following: [6]
   a) What is a static variable? How does it declare? How does it carry value?
   b) In the graphics mode, write the effect of following functions:
      void *linerel*(int dx, int dy);
      void *lineto*(int x, int y);
   c) What is a function prototype? When and where is it use?
   d) What is a data file? Write types of data files.

3. a) (i) Explain the function *initgraph*.
      (ii) Explain the steps involved to handle a data file in C.
   b) (i) Which operations one can perform on pointers? Explain with examples. [2]
      (ii) Explain with illustrations the difference between external variables and automatic variables. [2]
      (iii) float h, *j; int a, *b; are declarations. State why each of the following is invalid. [2]
      (1) b = j; | (2) b = &h; | (3) &a = b; | (4) h = j;

   OR

   b) (i) If char k[10][5]; is declared in a program, write the meaning of each of the following: [3]
      **k, *k+1, *(*k+1), *(k+1), *(*(k + 2) +3), *(k + 2) +3,
      (ii) Find the values of i, j and k after the execution of each of the following program segments [3]

(1)
```
int i = 5, j = 6, k = 7,*pi,*pj;
pj = &j;  j = ++k+12;  pi = pj;
i=*pi+7; k=*pi + 5; j = *pi +9;
```

(2)
```
int k = 5, i = 3, j = 7, *pi,*pj;
pi=&i; pj=&j;
*pi = *pj + 9; k = ++*pi + *pj/3 ;
j += *pj + 7; i = *pi + j;
```

P. T. O.

4. a) (i) Define a function to interchange the content of two variables. [2]

(ii) Define a function to find greatest common divisor (gcd) of two numbers. [2]

(iii) Discuss select sorting. [2]

b) (i) Write a C-program to solve $y' = xe^{-y}$, in the interval [0, 2) by Runge-Kutta method of order 2 given that $y(0) = 0.3$. [2]

(ii) Write a C-program to draw x-axis and y-axis and the graph of $y = e^x$, by considering center of the screen as the origin. [2]

(iii) Define a function which finds n! and using it, define a function C(n, r), where $C(n,r) = \frac{n!}{(n-r)!\,r!}$ for a C-program. [2]

OR

b) (i) Write a C-program to draw x-axis and y-axis and the circle of radius r having centre origin by using polar coordinates. [2]

(ii) Write a C- program to find $\int_0^1 e^x \sqrt{x}dx$ by using rectangle rule. [2]

(iii) Define the functions $h(x)=\sin(x)e^x$ and [2]

$$f(x,y)=\begin{cases} h(x+y)h(x*y) & if\ x \le y \\ h(x*y)/(h(x+y) & if\ x > y \end{cases}$$

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SEAT No.________

No of printed pages: 2

[80/A-44]

Sardar Patel University
Mathematics
M.Sc. Semester IV
Tuesday, 18 April 2017
2.00 p.m. to 5.00 p.m.
PS04EMTH01 - Problems and Exercises in Mathematics III

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) Let f be continuous on $[0,1]$. The value of $\lim_{n\to\infty}\int_0^1 f(x)\cos(2nx)dx$ is

(a) 0 (b) 1 (c) $\int_0^1 f(x)dx$ (d) $2\int_0^1 f(x)dx$

(2) Let f be the Cantor function on $[0,1]$. The value of $\int_0^1 f'(x)dx$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(3) The subspace topology on $\{(x,x) : x \in \mathbb{R}\} \subset \mathbb{R}_\ell \times \mathbb{R}_\ell$ is homeomorphic to _____ topology on $\mathbb{R}$.

(a) upper limit (b) standard (c) discrete (d) indiscrete

(4) Let $H = \{A \in M_n(\mathbb{R}) : \mathrm{tr}(A) = 0\}$. Then H is _____

(a) connected (b) compact (c) dense in $M_n(\mathbb{R})$ (d) open in $M_n(\mathbb{R})$

(5) The Galois group $G(\mathbb{Q}(\sqrt{2},\sqrt{3})/\mathbb{Q})$ is isomorphic to _____

(a) $\mathbb{Z}_4$ (b) $\mathbb{Z}_2$ (c) Klein group (d) none of these

(6) The number of onto homomorphisms from a field to itself which are not injective is _____

(a) 0 (b) 1 (c) infinitely many (d) none of these

(7) Let p be a nonconstant polynomial, and let $f(z) = \frac{1}{p(z)}$. Then ∞ is _____ of f.

(a) a removable singularity (c) a double pole
(b) a simple pole (d) an essential singularity

(8) Let f be an entire function vanishing on $\mathbb{Z} \setminus \{0\}$. Then the value of $f(0)$ is _____

(a) 0 (b) 1 (c) π (d) none of these

Q.2 Attempt any *Seven*. [14]

(a) Show that $\sin(x^2)$ is not uniformly continuous on $\mathbb{R}$.
(b) Show that every polynomial is a Lipschitz function on $[0,10]$.
(c) Find the total variation of $x^7 + x^2 - 2$ over $[1,2]$.
(d) Compute the residue of $f(z) = \frac{1}{1-e^z}$ at 0.
(e) Show that $z^7 - 4z^3 + z - 1 = 0$ has exactly three roots in $|z| = 1$.
(f) Determine the number of 3 dimensional subspaces of $\mathbb{Z}_3^4$.
(g) If p and q are distinct primes, then show that $\mathbb{Q}(\sqrt{p},\sqrt{q}) = \mathbb{Q}(\sqrt{p}+\sqrt{q})$.
(h) Let $\{A_\alpha\}$ be a collection of path connected subsets of a topological space X. If $\bigcap_\alpha A_\alpha \neq \emptyset$, then show that $\bigcup_\alpha A_\alpha$ is path connected.
(i) Show that a locally compact space need not be compact.

Q.3

(a) Let $f, g : [0,1] \to \mathbb{R}$ be $f(x) = x^3 \sin(\frac{1}{x})$ and $g(x) = x \sin(\frac{1}{x^3})$ if $x \neq 0$ and $f(0) = g(0) = 0$. Show that f is of bounded variation but g is not. [6]

(b) Show that there exist C^1 functions $u(x,y)$, $v(x,y)$ and $w(x,y)$ and $r > 0$ such that $u^5 + xv^2 - y + w = 0$, $v^5 + yu^2 - x + w = 0$, $w^4 + y^5 - x^4 - 1 = 0$ for all $(x,y) \in B((1,1);r)$ and $u(1,1) = v(1,1) = -w(1,1) = 1$. [6]

OR

(b) Find the volume of the largest parallelepiped contained in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where $a, b, c > 0$. [6]

Q.4

(c) Let $f : \mathbb{D} \to \mathbb{C}$ be analytic and $|f(z)| \leq 1$ for all $z \in \mathbb{D}$. Let $a \in \mathbb{D}$ and $f(a) = \alpha$. Show that $|f'(a)| \leq \frac{1-|\alpha|^2}{1-|a|^2}$. Deduce that there is no analytic function $f : \mathbb{D} \to \mathbb{D}$ satisfying $f(\frac{1}{2}) = \frac{3}{4}$ and $f'(\frac{1}{2}) = \frac{2}{3}$. State the results you use. [6]

(d) Let $\mathbb{H}_+ = \{z \in \mathbb{C} : \text{Im } z > 0\}$. Show that the map $\varphi(z) = \frac{z-i}{z+i}$ maps $\mathbb{H}_+$ onto the unit disc $\mathbb{D}$. Deduce that if f is an entire function and the range of f is contained in $\mathbb{H}_+$, then f is constant. [6]

OR

(e) Let f be a nonzero entire function. For $r > 0$, let $g(r) = \sup\{|f(z)| : |z| = r\}$. Show that g is a strictly increasing function. State the result you use. Is the stated result true if the domain is unbounded? Why? [6]

Q.5

(f) Let K be an extension of a field F. Define $G(K/F)$. If H is a subgroup of $G(K/F)$, then define the fixed field K_H. Let $K = \mathbb{Q}(2^{\frac{1}{3}}, \omega)$ and $F = \mathbb{Q}$, where $\omega^3 = 1$ and $\omega \neq 1$. Let $H = \{I, \sigma_1, \sigma_2\}$, where $\sigma_1(2^{\frac{1}{3}}) = 2^{\frac{1}{3}}\omega$, $\sigma_1(\omega) = \omega$, $\sigma_2(2^{\frac{1}{3}}) = 2^{\frac{1}{3}}\omega^2$ and $\sigma_2(\omega) = \omega$. Compute the fixed field K_H of H. [6]

(f) Let K be an extension of a field F, and let $a, b \in K$ be algebraic over F of degree m and n respectively. If $(m,n) = 1$, then show that $[F(a,b) : F] = mn$. State the results you use. [6]

OR

(g) (क) Show that the number of reducible polynomial over $\mathbb{Z}_p$ of the form $x^2 + ax + b$ is $\frac{p(p+1)}{2}$. [3]

(ख) Show that there is no vector space having 40 elements. [3]

Q.6

(h) Define linear continuum. Show that $[0,1] \times [0,1]$ with the dictionary order is a linear continuum. [6]

(i) Define second countable and separable topological spaces. Show that a metric space is second countable if and only if it is separable. [6]

OR

(h) Define a first countable topological space. Give an example of a topological space which is first countable but not second countable. Let X be a first countable topological space, $\emptyset \neq A \subset X$ and $x \in X$. Show that x is in the closure of A if and only if there is a sequence in A converging to x. [6]

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SEAT No.______ No. of Printed Pages : 2

[100/A-40]

Sardar Patel University

M.Sc.(Mathematics)(Sem-IV); Examination 2017;

PS04EMTH02 : Operator Theory;

15-04-2017; Saturday; Time-2.00 pm to 5.00 pm; Maximum Marks 70

Note: Notations and Terminologies are standard. H is a Hilbert space.

Q.1 Choose correct option from given four choices. [08]

(i) Let $x, y \in H$ such that $x \perp y$. Then

(a) $||x|| = ||x+y||$ (b) $||x|| \leq ||x+y||$ (c) $||x|| \geq ||x+y||$ (d) $||y|| \geq ||x+y||$

(ii) Let $E \subset H$ be a non-empty set. Then

(a) $H = E \oplus E^{\perp}$ (b) $E \cap E^{\perp} = \{0\}$ (c) $E^{\perp}$ is open (d) $E = E^{\perp\perp}$

(iii) Let $f : (\Omega, \mathcal{M}) \to \mathbb{C}$ be a bounded, measurable function. Let $\|f\|_\infty$ be the essential supremum norm and $\|f\|_\Omega$ be the usual supremum norm of f. Then

(a) $\|f\|_\infty = \|f\|_\Omega$ (b) $\|f\|_\infty \leq \|f\|_\Omega$ (c) $\|f\|_\infty \geq \|f\|_\Omega$ (d) none

(iv) Let $E : (\Omega, \mathcal{M}) \longrightarrow B(H)$ be a resolution of identity and $\omega_1, \omega_2 \in \mathcal{M}$. Then

(a) $E(\omega_1 \cup \omega_2) = E(\omega_1) + E(\omega_2)$
(b) $E(\omega_1 \setminus \omega_2) = E(\omega_1) - E(\omega_2)$
(c) $E(\omega_1 \cap \omega_2) = E(\omega_1)E(\omega_2)$
(d) $E(\omega_1 \setminus \omega_2) = E(\omega_1) + E(\omega_2)$

(v) Let $T \in B(H)$ be a normal operator. Then

(a) T has a polar decomposition
(b) T has a positive square root
(c) T has a unique polar decomposition
(d) none

(vi) Let $T \in B(H)$ be compact and normal, and let $f \in C(\sigma(T))$. Then $f(T)$ is

(a) compact (b) normal (c) self-adjoint (d) positive

(vii) Let $T \in B_1(H)$. Then

(a) $\|T\|_1 = \|T\|_2$ (b) $\|T\|_1 \leq \|T\|_2$ (c) $\|T\|_1 \geq \|T\|_2$ (d) none

(viii) Let $S \in B_1(H)$ and $T \in B_2(H)$. Then

(a) $ST \notin B_1(H)$ (b) $ST \notin B_2(H)$ (c) $S + T \in B_1(H)$ (d) $S + T \in B_2(H)$

Q.2 Attempt any **seven.** [14]

(i) Let $E \subset H$ be nonempty, closed, convex. Prove that E has an element of minimum norm.
(ii) Let $E : (\Omega, \mathcal{M}) \longrightarrow B(H)$ be a R.O.I., $\omega \in \mathcal{M}$, and $x \in H$. Prove that $E_{x,x}(\omega) = \|E(\omega)x\|^2$.
(iii) Let $E : (\Omega, \mathcal{M}) \longrightarrow B(H)$ be a R.O.I., $\omega, \omega' \in \mathcal{M}$ such that $\omega \subset \omega'$ and $E(\omega') = 0$. Then prove that $E(\omega) = 0$.
(iv) Let $T \in B(H)$ normal. Then prove that ever isolated point of $\sigma(T)$ is an eigenvalue.
(v) Let $T \in B(H)$ be self-adjoint such that $\sigma(T) \subset [0, \infty)$. Prove that T is a positive operator.
(vi) If $T \in B(H)$ is invertible, then prove that T has a polar decomposition.
(vii) Define trace-class operator and Hilbert-Schmidt operator.
(viii) Let $S, T \in B_2(H)$. Prove that $\|ST\|_2 \leq \|S\|_2 \|T\|_2$.
(ix) Prove that $4\langle x, y\rangle = \|x+y\|^2 - \|x-y\|^2 - i\|ix+y\|^2 + i\|ix-y\|^2 \ \ (x, y \in H)$.

(Continue on Page-2)

Q.3

(a) Let $x, y \in H$. Then prove the following: [6]
(i) $|\langle x, y\rangle| \leq \|x\|\|y\|$ and $\|x+y\| \leq \|x\|+\|y\|$; (ii) $x \perp y$ iff $\|y\| \leq \|\lambda x+y\|$ $(\lambda \in \mathbb{C})$.

(b) Let $T \in B(H)$. Then prove the following: [6]
(i) $N(T^*) = R(T)^{\perp}$ and $N(T) = R(T^*)^{\perp}$; (ii) T is normal iff $\|Tx\| = \|T^*x\|$ $(x \in H)$.

OR

(b) Let $M, N, T \in B(H)$, M and N be normal, and $MT = TN$. Prove that $M^*T = TN^*$. [6]

Q.4

(a) Let $T \in B(H)$ be normal. Then prove that there exists a unique resolution of identity $E : \mathcal{B}(\sigma(T)) \longrightarrow B(H)$ such that $T = \int_{\sigma(T)} \lambda dE(\lambda)$. [6]

(b) Let A be a unital, closed, normal subalgebra of $B(H)$. In standard notations, prove that there exists a R.O.I. E on $\mathcal{B}(\Delta)$ such that $T = \int_{\Delta} \widehat{T} dE$ $(T \in A)$. [6]

OR

(b) Let $T \in B(H)$ a normal operator. Then prove the following: [6]
(i) T is self-adjoint iff $\sigma(T) \subset \mathbb{R}$; (ii) T is unitary iff $\sigma(T) \subset \mathbb{T}$

Q.5

(a) Let $T \in B(H)$ be a normal operator and E be its spectral decomposition. Let $f \in C(\sigma(T))$ and $\omega_0 = f^{-1}(\{0\})$. Prove that $N(f(T)) = R(E(\omega_0))$. [6]

(b) Let $T \in B(H)$ be a positive operator. Prove that T has a unique positive square root. [6]

OR

(b) Let $M, N, T \in B(H)$. Let M and N be normal and T be invertible such that $M = TNT^{-1}$. Let $T = UP$ be the polar decomposition of T. Then prove that $M = UNU^{-1}$. [6]

Q.6

(a) Prove that $\|S+T\|_1 \leq \|S\|_1 + \|T\|_1$ $(S, T \in B_1(H))$. [6]

(b) Let $T \in B_1(H)$ and E be an orthonormal basis. Then prove that $\sum_{e \in E} |\langle Te, e\rangle| < \infty$ and $\sum_{e \in E} \langle Te, e\rangle$ is independent of E. [6]

OR

(b) Prove that $B_2(H)$ is an ideal in $B(H)$. [6]

THE END

[102/A-41] SEAT No.________

No of printed pages: 4

Sardar Patel University

M.Sc. (Sem-IV), PS04EMTH13, Financial Mathematics-II;
Saturday, 15^{th} April, 2017; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks; (iii) The required table of normal distribution is attached with this question paper.

1. A bank pays an interest rate of 12% per annum with continuously compounded. If \$100 is initially deposited, how much interest will be earned after one month ?
   (A) \$1.005 (B) \$101.005 (C) \$112 (D) none of these
2. If you put funds into an account that pays interest rate 9% compounded annually, approximately how many years does it take for your funds to quadruple ?
   (A) 9 (B) 20 (C) 16 (D) 18
3. The value of forward contract, F_0 is given by
   (A) $S_0e^{(r+r_f)T}$ (B) $S_0e^{(r-r_f)T}$ (C) $S_0e^{(r_f-r)T}$ (D) None of these
4. The Put Call Parity for European options on an asset paying dividend yield q is
   (A) $Se^{-q(T-t)} + P - C = Ke^{-r(T-t)}$ (B) $S + P + C = Ke^{-r(T-t)}$
   (C) $S + P - C = Ke^{-r(T-t)}$ (D) $Se^{-q(T-t)} - P + C = Ke^{-r(T-t)}$
5. A short position in − − − − − option should be hedged by a long position of delta number of underlying assets.
   (A) American put (B) European put
   (C) European call (D) None of these
6. The value of theta at striking price is always
   (A) maximum (B) minimum (C) 0 (D) none of these
7. The variance of a random variable X is given by
   (A) $V(X) = E(X^2) - [E(X)]^2$ (B) $V(X) = [E(X)]^2 - E(X^2)$
   (C) $V(X) = E(X) - E(X^2)$ (D) None of these
8. The probability of a 'down movement' in one step binomial model is given by
   (A) $\frac{e^{rT}-d}{u-d}$ (B) $\frac{e^{rT}-d}{d-u}$ (C) $\frac{e^{rT}-u}{d-u}$ (D) $\frac{e^{-rT}-d}{u-d}$

Q.2 Attempt any *seven*: [14]

(a) If you can afford a ₹500 monthly car payment at the end of the month, how much amount of car can you afford if interest rate is 7 % per annum with monthly compounding for 36 months loan?
(b) Give a formula that approximates the number of years it would take your funds to triple if you received interest at a rate r compounded continuously.
(c) Define rate of return with an example.
(d) Write down BSM formulas for currency options.
(e) Define the greek letter rho. What is rho for European call option?
(f) Write down formulas of theta for European options.
(g) Write down relationship between Δ, Γ and Θ using BSM differential equation on an asset paying no dividend.
(h) Draw tree of asset prices with four equal time intervals.
(i) Define delta using binomial tree.

Q.3

(a) Define yield curve and show that the yield curve $\bar{r}(t)$ is a non decreasing function of t iff $(P(t))^{\alpha} \leq P(\alpha t)$ for all $0 \leq \alpha \leq 1, t \geq 0$. [6]

(b) A company needs a certain type of machine for the next five years. They presently own such a machine, which is now worth £8000 but will lose £2000 in value in each of the next four years, after which it will be worthless and unusable. The (beginning of the year) value of its yearly operating cost is £9000, with this amount expected to increase by £2000 in each subsequent year that is used. A new machine can be purchased at the beginning of i^{th} year for a fixed cost of £$(2000(10+i))$, where $i = 1, 2, 3, 4, 5$. Its value decreases by £2000 per year of use. The operating cost of a new machine is £6000 in its first year, with an increase of £1000 in each subsequent year. If the interest rate is 10% compounded yearly, when should the company purchase a new machine? [6]

OR

(b) A professor who plans to retire in 20 in years has decided to put an amount C in the bank at the beginning of the each of the next 240 months, after which he will withdraw £3000 at the beginning of each of the next 360 months. The nominal interest rate is 6% per annum with compounded monthly. Find the value of C.

Q.4

(a) Discuss jump condition for discrete dividend. [6]

(b) Derive the BSM formulas of an asset providing a constant dividend yield. [6]

OR

(b) If the 2-year interest rates in Australia and United States are 5% and 7% per annum with continuously compounding, respectively, and the spot exchange rate between Australian dollar (AUD) to the US dollar(USD) is 0.6200 USD per AUD. Find the value of futures contract. Justify it.

Q.5

(a) Explain: 'a short position in a European put option should be hedged by a short position of underlying assets' with an example. [6]

(b) Derive Gamma for European put option on an asset paying no dividend. [6]

OR

(b) Explain Generalized two step binomial model.

Q.6

(a) Derive formula for American put option using binomial model in which there are N subintervals of length Δt. [6]

(b) A 2 months American put option on a stock has an exercise price of \$480. The current stock price is \$484, the risk free interest rate is 10% per annum with continuously compounding, the dividend yield on the stock is 3% per annum and the volatility is 25% per annum. Using binomial model, find the value of option when the life of the option is divided into 4 subintervals of length half month. [6]

OR

(b) Determine p, u and d in terms of σ, r and Δt using binomial model.

* * * * *

Table for $N(x)$ When $x \geqslant 0$

This table shows values of $N(x)$ for $x \geqslant 0$. The table should be used with interpolation. For example,

$$\begin{aligned} N(0.6278) &= N(0.62) + 0.78[N(0.63) - N(0.62)] \\ &= 0.7324 + 0.78 \times (0.7357 - 0.7324) \\ &= 0.7350 \end{aligned}$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for $N(x)$ When $x \leqslant 0$

This table shows values of $N(x)$ for $x \leqslant 0$. The table should be used with interpolation. For example,

$$\begin{aligned} N(-0.1234) &= N(-0.12) - 0.34[N(-0.12) - N(-0.13)] \\ &= 0.4522 - 0.34 \times (0.4522 - 0.4483) \\ &= 0.4509 \end{aligned}$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

— x —

SEAT No.________ NO. OF PRINTED PAGES: 2

[341/A-33]

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-IV) Examination
April – 2017
Thursday, 20 April, 2017
Time: 02:00 PM to 05:00 PM

Subject: Mathematics **Course No.: PS04EMTH15 (Relativity-II)**

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.
(3) Use the expressions in the Appendix if necessary.

Q-1 Choose most appropriate answer from the options given. (08)

(1) Which one of the following is correct?
(a) $g^{ij} = -g^{ji}$ (b) $g^{ij} = G^{ij}$ (c) $g^i_j = \delta^i_j$ (d) none of these

(2) Which one of the following is true?
(a) $\Gamma^i_{jk} = \Gamma^i_{kj}$ (b) Γ^i_{jk} form a tensor type (2,1)
(c) Γ^i_{jk} form a tensor of type (1,2) (d) none of these

(3) The condition for a Riemannian space to be empty is ________
(a) $R_{hijk} = 0$ (b) $R_{ij} = 0$ (c) $R_{ij} \neq 0$ (d) $R_{hijk} \neq 0$

(4) Which one of the following is correct?
(a) Schwarzschild exterior solution is valid in empty region.
(b) Schwarzschild interior solution is valid in empty region.
(c) Schwarzschild exterior solution represents correct model of the universe.
(d) none of the above

(5) In Schwarzschild metric $r = 0$ ______________
(a) is a removable singularity (b) always lies inside the body
(c) is a regular point (d) is an essential singularity

(6) Bending of light ______________.
(a) can not be explained using general relativity
(b) can not be observed
(c) can not be explained using Newtonian theory
(d) can be explained using Newtonian theory

(7) Which one of the following is correct?
(a) Acceleration of a particle moving in de-Sitter universe is zero
(b) Acceleration of a particle moving in Einstein universe is zero
(c) de-Sitter universe is non-static
(d) Einstein universe is non-static

(8) Which one of the following is correct?
(a) In Robertson-Walker metric R is a decreasing function
(b) In Robertson-Walker metric R is an increasing function
(c) Robertson-Walker metric describes an empty space
(d) None of the above.

Q-2 Answer any **Seven.** (14)

(1) State expressions of Christoffel symbols of both kinds.
(2) State symmetry properties of Ricci tensor and hence find the maximum number of independent components of Ricci tensor.
(3) State Schwarzschild exterior metric in Kruskal coordinates.
(4) State Birkhoff's theorem.
(5) What is condition for dust in the energy-momentum tensor for a perfect fluid.

(6) What is volume of Einstein's spherical universe?
(7) Is it true that de-Sitter universe is completely empty? Justify your answer.
(8) State cosmological principle for the universe.
(9) What is meant by Friedmann models?

Q-3

(a) State the expression of Riemann tensor and discuss its algebraic properties. (06)
(b) Obtain geodesic equations on the unit sphere. (06)

OR

(b) Define covariant derivative of a covariant vector. Show that it forms a covariant tensor of rank two.

Q-4

(a) State Schwarzschild exterior metric. Find the expression of velocity of a radial light ray. In terms of this describe the singularity at $r = 2m$. (06)
(b) Give outlines of derivation of Schwarzschild interior metric. (06)

OR

(b) Derive the relativistic equation of orbit of a planet.

Q-5

(a) Giving all details derive the metric of de-Sitter universe. (06)
(b) Discuss the motion of a particle in Einstein universe. (06)

OR

(b) Define Einstein space and show that Einstein universe is not an Einstein space.

Q-6

(a) For the Robertson-Walker metric derive the relation between the scale factor and red-shift. (06)
(b) Obtain metric for a 3-sphere in the form $d\sigma^2 = R^2\left[\frac{d\bar{r}^2}{1-\bar{r}^2} + \bar{r}^2 d\Omega^2\right]$, the notations being usual. (06)

OR

(b) Discuss in brief the Freidmann models of the universe.

Appendix

For the spherically symmetric static metric $ds^2 = -e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu(r)}dt^2$, non-zero independent Christoffel symbols and components of Ricci tensor are as under,

$\Gamma^1_{11} = \frac{\lambda'}{2}$, $\Gamma^1_{22} = -re^{-\lambda}$, $\Gamma^1_{33} = -re^{-\lambda}\sin^2\theta$, $\Gamma^1_{44} = e^{\nu-\lambda}\frac{\nu'}{2}$, $\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r}$, $\Gamma^2_{33} = -\sin\theta\cos\theta$, $\Gamma^3_{23} = \cot\theta$, $\Gamma^4_{14} = \frac{\nu'}{2}$

$R_{11} = \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} - \frac{\lambda'}{r}$, $R_{22} = -1 + e^{-\lambda} - \frac{1}{2}re^{-\lambda}(\lambda' - \nu')$,

$R_{33} = \sin^2\theta\, R_{22}$, $R_{44} = -e^{\nu-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} + \frac{\nu'}{r}\right]$

For R-W metric the non-zero components of Ricci tensor are given by,

$$\frac{R_{11}}{g_{11}} = \frac{R_{22}}{g_{22}} = \frac{R_{33}}{g_{33}} = \frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}^2+k}{R^2}\right), \quad \frac{R_{44}}{g_{44}} = \frac{3\ddot{R}}{R}$$

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SEAT No.________

[22]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-IV), PS04EMTH21, Mathematics Education-II;
Saturday, 22^{nd} April, 2017; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. $\text{Log}(-1) =$
   (A) ∞ (B) $i\pi$ (C) $-i\pi$ (D) $-\infty$
2. $(\frac{1-i}{\sqrt{2}})^{20} =$
   (A) 1 (B) $\frac{1}{2}$ (C) -1 (D) none of these
3. An investor invest ₹100 with the interest rate 10% per annum with continuously compounding. Then the amount after one year will be
   (A) ₹110 (B) ₹90 (C) ₹100 (D) none of these
4. The set of discontinuities of the function $f : \mathbb{R} \to \mathbb{R}$,
$$f(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q} \\ 0, & \text{otherwise,} \end{cases}$$
   (A) $\mathbb{Q}$ (B) $\mathbb{R}$ (C) $\{0, 1\}$ (D) $\{\ \}$
5. The equation $x^2 + y^2 + 2x - 2 = 0$ represents
   (A) ellipse (B) parabola (C) circle (D) hyperbola
6. Which one from the following is an algebraic number over $\mathbb{Q}$?
   (A) e^{π} (B) e (C) π (D) $\sin(\pi)$
7. The Bieberbach conjecture was proved by
   (A) Euler (B) Louis de Branges (C) Apple and Haken
   (D) Paul Erdos
8. Gauss conjectured that the number of primes less than or equal to n is approximately
   (A) $n\ln(n)$ (B) $\frac{\ln(n)}{n}$ (C) $\frac{n}{\ln(n)}$ (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Draw graph of $f(x) = |5 - x|$.
(b) Transform $r = 2(1 + \cos\theta)$ in cartesian form.
(c) Give postulates of Coordinate Geometry.
(d) The daily profit of sugar refinery is given by $P(x) = 8x - 0.02x^2$ where x is the number of tons of sugar refined. How many tons will give maximum profit and what is the maximum profit ?
(e) Evaluate $\lim\limits_{n\to\infty} \frac{(1+2+\cdots+n)^2}{(n^2+1)(1-n^2)}$.
(f) Who was considered to be the father of set theory? What was his nationality ?
(g) What is Bolzano's paradox ?
(h) What is four color problem ?
(i) Discuss Riemann's hypothesis.

Q.3

(a) Write biography from the following (**any one**): [06]
(i) John Napier (ii) Fermat (iii) René Descartes

(b) Give applications of Coordinate Geometry. [06]

OR

(b) If $60^x = 3$ and $60^y = 5$, then find $12^{\frac{1-x-y}{2(1-y)}}$.

Q.4

(a) Write biography from the following (**any one**): [06]
(i) Archimedes (ii) Leibnitz (iii) Newton

(b) Discuss in detail the development of calculus. [06]

OR

(b) Find the extremum of $f(x, y) = x^2 + y^2 + 2(\frac{x+y}{xy})$.

Q.5

(a) State (i) Zeno's paradox (ii) Euler's paradox (iii) Russell's paradox. [06]

(b) Discuss Cantor's contribution in set theory. [06]

OR

(b) Five children A,B,C,D and E participated in the race. From the following pairs of statements, exactly one is statement is true. Find the rank of each child in the race.
(I) A is second, B is third
(II) C is third, D is fifth
(III) D is first, C is second
(IV) B is first, E is fourth

Q.6

(a) Discuss the development of modern mathematics. [06]

(b) State (i) Ramsey's problem (ii) Konigsberg seven bridge problem. [06]

OR

(b) Discuss prime number theorem.

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[23]
A18

SEAT No.________ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-IV) Examination
April – 2017
Saturday April 22, 2017
Time: 02:00 PM to 5:00 PM
Subject: Mathematics
Course No.PS04EMTH30 (Operations Research)

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) An LPP must have
(a) a linear objective function (b) at least one linear constraints
(c) at least three decision variables (d) all of these

(2) A constraint puts restriction on
(a) value of objective function (b) use of resources
(c) uncertainty of solution (d) none of these

(3) In the graphical method feasible region is ________.
(a) a parallelogram (b) convex (c) open (d) a null set

(4) A basic feasible solution to a system is called ____________ if any of basic variables vanishes.
(a) non-degenerate (b) non-basic
(c) degenerate (d) non-appropriate

(5) Which one of the following is correct?
(a) transportation problem is an assignment problem
(b) assignment problem is a transportation problem
(c) transportation problem is an NLPP
(d) none of these

(6) Which of the following term does not occur in a transportation problem?
(a) resources (b) availability (c) supply (d) activity

(7) Which of the following methods cannot be used to sole an NLPP?
(a) Method of Lagrange's multipliers (b) Big M method
(c) Graphical method (d) all of these

(8) In a non-linear programming problem
(a) the objective function is non-linear
(b) one or more constraints are non-linear
(c) both (a) and (b)
(d) either (a) or (b)

Q-2 Answer any **Seven.** (14)

(1) What is the meaning of unbounded solution?
(2) Show that the set $\{(x, y) \in R^2 : 0 \le x \le 1, 0 \le y \le 1\}$ is a convex set.
(3) What is use of a slack variable?
(4) How a minimization problem is dealt in simplex method ?
(5) Define net evaluation in simplex method.
(6) Explain the term "dummy destination" in context of a transportation problem.
(7) Describe NWC method for finding an IBFS to a transportation problem.
(8) Explain constraints in an assignment problem.
(9) Explain drawbacks of using graphical method for solving an NLPP.

Q-3

(a) Describe the steps in graphical method for solving an LPP. Also describe limitations of this method. (06)

(b) A company produces two different headache pills A and B. A pill of type A contains 2 mg aspirin, 5 mg bicarbonate and 1 mg codeine. A pill of type B contains 1 mg aspirin, 8 mg bicarbonate and 6 mg codeine. Research shows that a patient of headache requires at least 12 mg aspirin, 24 mg codeine and74 mg bicarbonate for the immediate relief. Formulate a standard LPP to determine the least number of pills a patient should take for immediate relief. (06)

OR

(b) Solve the following L.P.P. by Simplex Method :
Max $Z = 5x_1 + 10x_2 + 8x_3$ subject to
$3x_1 + 5x_2 + 2x_3 \leq 60$, $4x_1 + 4x_2 + 4x_3 \leq 72$, $2x_1 + 4x_2 + 5x_3 \leq 100$
and $x_1, x_2, x_3 \geq 0$.

Q-4

(a) When Big M method is useful? Describe the steps in it. (06)

(b) Obtain the dual of the following problem : (06)
Max $Z = x_1 + 3x_2 - 2x_3 + 5x_4$
subject to $3x_1 - x_2 + x_3 - 4x_4 = 2$, $5x_1 + 3x_2 - x_3 - 2x_4 = 3$;
$x_1, x_2 \geq 0$ and x_3, x_4 unrestricted in sign.

OR

(b) Solve the following L.P.P. by using two-phase method :
Max $Z = 5x_1 + 3x_2$
subject to $2x_1 + x_2 \leq 1$, $x_1 + 4x_2 \geq 6$, and $x_1, x_2 \geq 0$.

Q-5

(a) Describe uv-method for determining optimality for a transportation problem. (06)

(b) Obtain an initial BFS to the following TP using VAM. Also find the cost involved in the solution you obtained. (06)

	D1	D2	D3	D4	D5	Availability
O1	4	5	7	9	10	20
O2	3	1	2	6	9	30
O3	8	12	15	30	4	17
O4	3	2	10	13	17	13
Demand	40	8	7	19	6	

OR

(b) Solve the following assignment problem using Hungarian method.

	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

Q-6

(a) Derive the condition for maximization in terms of bordered Hessian matrix for a general NLPP with m (<n) constraints. (06)

(b) Obtain the condition for Maximum $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$ subject to $x_1 + x_2 + x_3 = 15$, $2x_1 - x_2 + 2x_3 = 20$; $x_1, x_2, x_3 \geq 0$. (06)

OR

(b) Find the measures of sides parallel to coordinate planes of a rectangular parallelepiped with largest volume inscribed in the unit sphere.

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