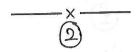
SARDAR PATEL UNIVERSITY
M.Sc. (Mathematics) Semester - I Examination

DS0	Saturday, 03 rd	November, 2018		
Time: 10:00 a.m.	to 01:00 p m	natical Classical N	fechanics	
	_	of the very t	Maximum marks: 70	
(2) Assume u	sual/standard notations	wherever applicable.	estion.	
Q-1 Choose the mos	t appropriate option fo	or each of the followi	ng questions.	[08]
1. A particle mo	ving inside a circle has	s no constra	int.	
(a) holonomic	(b) non-holono	mic (c) scleronomic	d) rheonomic	
2. Degrees of fre	edom of a particle mov	$ying on x^2 + y^2 = r^2,$	r constant, is	
(a) 0	(b) 1	(c) 2	0 mal(d) 3 m , har stere	
3 is the	e solution for a curve o	of minimum surface a	rea of revolution.	
(a) Catenary	(b) Cycloid	(c) Straight lin	e (d) Great circle	
4. If the compon is cor	ent of total force $ar{F}$ al	ong \hat{n} is zero, then a	long \hat{n} the component of	
$\overline{(a)} \ \overline{p}$	(b) T	/ \	(d) <i>h</i>	
	linates of a system are			
(a) $-L$	(b) L	(c) H	(d) -H	
6. From Hamilton	n's equations of motion	n we have $I \frac{\partial H}{\partial I} I' -$	What quadrins a <u>n comen</u>	
(a) $\dot{\eta}J$	(b) J	(c) $-\dot{n}J$	(d) <u>∂H</u>	
7. For symplectic plectic.	matrix M and $J = ($	$\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, the matrix	need not be sym-	
(a) <i>MJM'</i>	(b) MJM^2	(c) J'M	(d) $M + M'$	
$8. [a_0, n_0] - \{n_0, a_0\}$	ol - · notation	aa baimm1	40.5	
(a) 2	(b) -2	(c) -1	(d) 0	
Q-2 Attempt any ser	ven of the following.			[a II]
			Prove that a coordinate t	[14]
(b) State principl	e of virtual work	give its example.		
(c) State the con	dition for contrary	Cultural of CV2 of	Talka Panege utem	
	dition for extremum of			
it conserved?	lized momentum conju	gate to a generalized	coordinate q_j . When is	
(e) Show that if I	Hamiltonian does not o	depend on q_j explicit	ly, then it is conserved.	
	on's modified principle.			
(g) State the sym	plectic condition for a	transformation to be	e canonical.	
	nical transformation.			
2000 8550	dentity for Poisson bra	ickets.		
	= =	(T)	(P.TO)

- Q-3 (a) State D'Alembert's principle and derive Lagrange's equations of motion in general [06] form from it. (b) Describe an atwood machine. Stating its constraints derive its Lagrangian. [06](b) Define a rigid body. Describe the constraints and degrees of freedom in a rigid [06]body. Q-4 (a) Define action integral. State Hamilton's principle and hence derive Lagrange's [06] equations of motion from it. (b) Let $L=e^{\gamma t}\left(\frac{m\dot{q}^2}{2}-\frac{kq^2}{2}\right)$ be Lagrangian of a system. Compute energy function [06]and the generalized momentum conjugate to q. Is any of them conserved? Justify. (b) State and prove the law of conservation of angular momentum in Lagrangian [06]formalism. Q-5 (a) Using an appropriate Legendre transformation derive Hamilton's equations of [06]motion from Lagrange's equations of motion. [06](b) Lagrangian for a system can be written as $L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2 + y^2},$ where, a, b, c, f, g and k are constants. Compute a corresponding Hamiltonian. What quantities are conserved? OR (b) A Hamiltonian of one degree of freedom has the form $H = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{ba}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2},$ where a, b, α , and k are constants. Find a corresponding Lagrangian. Is Hamiltonian conserved? Justify. Q-6 (a) Prove that a coordinate transformation is canonical if and only if all Poisson [06]brackets are invariant under the transformation. i. Define Poincaré integral. Show that it is a canonical invariant. [03]ii. Using fundamental Poisson brackets find the values of α and β for which the equations $Q = q^{\alpha} \cos \beta p, \ P = q^{\alpha} \sin \beta p$ represent a canonical transformation.

 - (b) For one dimensional system with Hamiltonian $H = \frac{p^2}{2} \frac{1}{2a^2}$, show that there is a constant of motion $D = \frac{pq}{2} - Ht$.



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