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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Saturday, 03rd November, 2018

PS01EMTH22, Mathematical Classical Mechanics

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: (1) Figures to the right indicate marks of the respective question.
(2) Assume usual/standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions. [08]

1. A particle moving inside a circle has no ______ constraint.
   (a) holonomic (b) non-holonomic (c) scleronomic (d) rheonomic
2. Degrees of freedom of a particle moving on $x^2 + y^2 = r^2$, r constant, is ______.
   (a) 0 (b) 1 (c) 2 (d) 3
3. ______ is the solution for a curve of minimum surface area of revolution.
   (a) Catenary (b) Cycloid (c) Straight line (d) Great circle
4. If the component of total force $\bar{F}$ along $\hat{n}$ is zero, then along $\hat{n}$ the component of ______ is conserved.
   (a) $\bar{p}$ (b) T (c) V (d) h
5. If all the coordinates of a system are ignorable, then Routhian $R =$ ______.
   (a) $-L$ (b) L (c) H (d) $-H$
6. From Hamilton's equations of motion, we have $J\frac{\partial H}{\partial \eta}J' =$ ______.
   (a) $\dot{\eta}J$ (b) J (c) $-\dot{\eta}J$ (d) $\frac{\partial H}{\partial \eta}$
7. For symplectic matrix M and $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, the matrix ______ need not be symplectic.
   (a) MJM' (b) MJM^2 (c) $J'M$ (d) $M + M'$
8. $[q_2, p_2] - \{p_2, q_2\} =$ ______; notations being usual.
   (a) 2 (b) -2 (c) -1 (d) 0

Q-2 Attempt *any seven* of the following. [14]

(a) Define a rheonomic constraint and give its example.

(b) State principle of virtual work.

(c) State the condition for extremum of the integral $\int_{y_1}^{y_2} f(x, \dot{x}, y)\, dy$?

(d) Define generalized momentum conjugate to a generalized coordinate q_j. When is it conserved?

(e) Show that if Hamiltonian does not depend on q_j explicitly, then it is conserved.

(f) State Hamilton's modified principle.

(g) State the symplectic condition for a transformation to be canonical.

(h) Define a canonical transformation.

(i) State Jacobi identity for Poisson brackets.

(P.T.O)

Q-3 (a) State D'Alembert's principle and derive Lagrange's equations of motion in general form from it. [06]

(b) Describe an atwood machine. Stating its constraints derive its Lagrangian. [06]

OR

(b) Define a rigid body. Describe the constraints and degrees of freedom in a rigid body. [06]

Q-4 (a) Define action integral. State Hamilton's principle and hence derive Lagrange's equations of motion from it. [06]

(b) Let $L = e^{\gamma t}\left(\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\right)$ be Lagrangian of a system. Compute energy function and the generalized momentum conjugate to q. Is any of them conserved? Justify. [06]

OR

(b) State and prove the law of conservation of angular momentum in Lagrangian formalism. [06]

Q-5 (a) Using an appropriate Legendre transformation derive Hamilton's equations of motion from Lagrange's equations of motion. [06]

(b) Lagrangian for a system can be written as [06]

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2+y^2},$$

where, a, b, c, f, g and k are constants. Compute a corresponding Hamiltonian. What quantities are conserved?

OR

(b) A Hamiltonian of one degree of freedom has the form [06]

$$H = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{ba}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha}t) + \frac{kq^2}{2},$$

where $a, b, \alpha,$ and k are constants. Find a corresponding Lagrangian. Is Hamiltonian conserved? Justify.

Q-6 (a) Prove that a coordinate transformation is canonical if and only if all Poisson brackets are invariant under the transformation. [06]

(b) i. Define Poincaré integral. Show that it is a canonical invariant. [03]

ii. Using fundamental Poisson brackets find the values of α and β for which the equations [03]

$$Q = q^\alpha \cos \beta p, \ P = q^\alpha \sin \beta p$$

represent a canonical transformation.

OR

(b) For one dimensional system with Hamiltonian $H = \frac{p^2}{2} - \frac{1}{2q^2}$, show that there is a constant of motion $D = \frac{pq}{2} - Ht$. [06]

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