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SEAT NO._____

No. of printed pages 2

Sardar Patel University

M.Sc. (Mathematics) I[st] Semester

PS01EMTH21 - Graph Theory I

Saturday, November 3, 2018

Time: 10.00 a.m. to 01.00 p.m. **Total Marks: 70**

Q.1 Choose the most appropriate option in the following questions. [08]

1. For $G = C_7$ if $\text{diam}(G) = d$ and $\text{rad}\ (G) = r$, then

   (a) $d = r$ (b) $d < r$ (c) $d > r$ (d) None of these

2. If G is complete symmetric diagraph with n vertices,then $|E(G)| =$

   (a) $\frac{n(n-1)}{2}$ (b) $n(n-1)$ (c) n^2 (d) None of these

3. Let T be a spanning in-tree with root R. then

   (a) $d^+(R) > 0$ (b) $d^-(R) = 0$ (c) $d^-(R) > 0$ (d) None of these

4. A regular diagraph is

   (a) weakly connected (b) Euler (c) symmetric (d) None of these

5. The chromatic number of C_{2m+1} $(m \in \mathbb{N})$ is

   (a) 2 (b) m (c) $2m$ (d) None of these

6. The coefficient c_4 in chromatic polynomial of K_5 is

   (a) 0 (b) 1 (c) 5 (d) None of these

7. If $G = C_6$ and M is a maximal matching in G, then $|M| =$

   (a) 2 (b) 3 (c) 4 (d) None of these

8. If $G = P_9$, then $(\beta(G), \beta'(G)) =$

   (a) $(5,5)$ (b) $(4,5)$ (c) $(4,4)$ (d) None of these

Q.2 Attempt any seven. [14]

1. Find the radius of $K_{m,n}$ $(m, n \geq 2)$.
2. Write any four properties of tree.
3. Define spanning out-tree with example.
4. Define strongly connected diagraph with example.
5. If G is a tree, then show that $\chi(G) = 2$.
6. What is four color problem?
7. Is K_5 uniquely colorable? Justify your answer.
8. State Hall's theorem.
9. Define vertex cover with example.

Q.3

(a) Discus in details Teleprinter problem for $r = 4$. [06]

(b) Define the following with example (1) Clique, (2) complete bipartite graph and (3) Independence number [06]

(P.T.O)

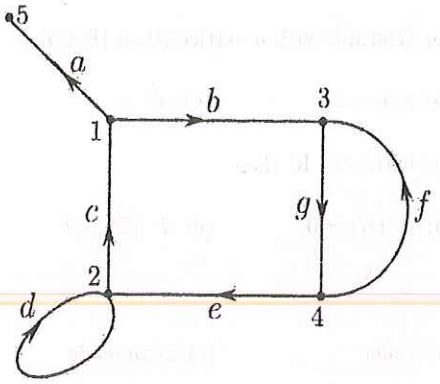
OR

(b) Define the following with example (1) Eccentricity of vertex, (2) Center of graph and (3) Radius of graph. [06]

Q.4

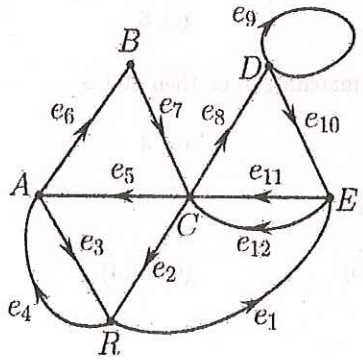
(a) Let G be a diagraph without self loop. Let A and B denote the incidence matrix and circuit matrix respectively. Suppose that the edges are arranged in the same order in A and B. Show that $AB^T = 0$. [06]

(b) Define Adjacency matrix for diagraph. Find Adjacency matrix of the following diagraph. [06]



OR

(b) Define spanning in-tree. In the following diagraph, find a spanning in-tree having the vertex R as a root. [06]



Q.5

(a) Find Chromatic polynomial of $K_{2,2}$. [06]

(b) Suppose G be a simple graph with n vertices ($n \geq 3$). If $\delta(G) \geq \frac{n}{2}$, then show that G is Hamiltonian. [06]

OR

(b) If G has a Hamiltonian cycle, Then show that for each nonempty set $S \subseteq V$, the graph $G \setminus S$ has at most $|S|$ components. [06]

Q.6

(a) If G is a bipartite graph, then Show that $\alpha'(G) = \beta(G)$. [06]

(b) Define symmetric difference. Show that every component of the of the symmetric difference of two matchings is a path or an even cycle. [06]

OR

(b) For $k > 0$, show that every k–regular bipartite graph has a perfect matching. [06]