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SEAT No.________

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Sardar Patel University

M.Sc. (Sem-I), PS01CMTH05, Methods of Differential Equations;
Thursday, 01^{st} November, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of differential equation $(y'')^2 + y^3y' = 0$ is
   (A) 0 (B) 2 (C) 3 (D) 1
2. The set of singular points of $xy'' + \sin xy' + xy = 0$ is
   (A) $\{0\}$ (B) φ (C) $\{1\}$ (D) none of these
3. $\Gamma(\frac{1}{2}) =$
   (A) $\sqrt{\pi}$ (B) $-\sqrt{\pi}$ (C) 1 (D) none of these
4. $\int_{-1}^{1} xP_4(x)dx =$
   (A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{15}$ (D) none of these
5. Which of the following is an integrating factor of $ydx + xdy$?
   (A) $\frac{1}{y}$ (B) $\frac{1}{x}$ (C) $\frac{1}{y^2}$ (D) none of these
6. Which one is not a homogeneous Pfaffian differential equation?
   (A) $x^2dx + y^2dy + 2zydz = 0$
   (B) $(x^2 - 1)dx + (y^2 - 1)dy + (z^2 - 1)dz = 0$
   (C) $x^2ydx + y^2zdy + z^2xdz = 0$
   (D) none of these
7. $F(1, \frac{1}{2}; \frac{1}{2}; -1) =$
   (A) 2 (B) 1 (C) -1 (D) none of these
8. $F(\alpha, \beta; \gamma; 0)=$
   (A) 0 (B) -1 (C) $\frac{\alpha\beta}{\gamma}$ (D) 1

Q.2 Attempt any ***seven***: [14]

(a) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)} x^n$.
(b) Verify the analyticity of $xy'' + (e^x - 1)y' + xy = 0$ at 0.
(c) Show that between any two positive roots of J_1 there is a root of J_0.
(d) Define gamma function.
(e) State Fourier Legendre expansion theorem.
(f) State Picard's theorem.
(g) Find a partial differential equation by eliminating f and g from $z = f(x+y) + g(x-y)$.
(h) Show that $xF(1, 1; 2; x) = -\ln(1 - x)$.
(i) Find radius of convergence of Gauss's hypergeometric series.

[P.T.O.]

Q.3

(a) Solve: $y'' + (x-1)y' + y = 0$ near 1. [6]

(b) Solve: $2x^2y'' + 3xy' - (x+1)y = 0$ near 0. [6]

OR

(b) Solve: $x^5y'' + 2x^4y' - y = 0$ near ∞.

Q.4

(a) State and prove orthogonality of Legendre's polynomials. [6]

(b) Prove: $x = \sum\limits_{n=1}^{\infty} \dfrac{2}{\lambda_n J_2(\lambda_n)} J_1(\lambda_n x)$, $x \in (0,1)$, where $\{\lambda_n\}$ is a sequence of positive roots of J_1. [6]

OR

(b) Prove: $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum\limits_{n=-\infty}^{\infty} J_n(x)\, t^n$.

Q.5

(a) Solve $y' = x + y$, $y(0) = 1$ using Picard's method of successive approximations. [6]

(b) Find a necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $F(u,v) = 0$ not involving x or y explicitly. [6]

OR

(b) Verify that the differential equation $x(y^2-1)dx + y(x^2-z^2)dy - z(y^2-1)dz = 0$ is integrable and find its primitive.

Q.6

(a) State and prove integral representation of Gauss's hypergeometric function. [6]

(b) Solve: $(p^2+q^2)y = qz$ using Charpit's method. [6]

OR

(b) Solve: $z^2 = pqxy$ using Jacobi's method.