SC

Seat No. ________

[39]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Saturday, 03rd November, 2018

PS01EMTH02, Mathematical Classical Mechanics

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: (1) Figures to the right indicate marks of the respective question.
(2) Assume usual/standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions. [08]

1. A force $\bar{F}$ is called conservative if ______ for some scalar potential V.
   (a) $\bar{F} = \lambda V$ (b) $\bar{F} = -\nabla \times V$ (c) $\bar{F} = \nabla V$ (d) $\bar{F} = -\nabla V$
2. Degrees of freedom of a particle moving on a circle is ______.
   (a) 3 (b) 2 (c) 1 (d) 0
3. ______ is a brachistochrone curve.
   (a) Catenary (b) Cycloid (c) Straight line (d) Great circle
4. If Lagrangian does not depend on t explicitly, then ______ is conserved.
   (a) L (b) h (c) q (d) p
5. If all the coordinates of a system are non-cyclic, then Routhian $R =$ ______.
   (a) L (b) H (c) $-L$ (d) $-H$
6. Hamilton's equations of motion in matrix form is given by $J\frac{\partial H}{\partial \eta} =$ ______.
   (a) J (b) $\dot{\eta}J$ (c) $\dot{\eta}$ (d) $-J\dot{\eta}$
7. For symplectic matrices M and N, ______ need not be symplectic.
   (a) MN' (b) $M+N$ (c) $M^{-1}N$ (d) MJN
8. $[q_1, p_2] + \{p_2, q_1\} =$ ______; notations being usual.
   (a) 0 (b) -1 (c) -2 (d) 2

Q-2 Attempt *any seven* of the following. [14]

(a) Define a scleronomic constraint and give its example.

(b) State Lagrange's equations of motion in presence of frictional forces.

(c) State the condition for extremum of the integral $\int_{x_1}^{x_2} f(y_1, \dots, y_n, \dot{y}_1, \dots, \dot{y}_n, x)\, dx$?

(d) State law of conservation of linear momentum in Lagrangian formalism.

(e) Let $L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2+y^2}$ be Lagrangian of a system. How many generalized coordinates are there? Which of them are cyclic? Justify.

(f) State Hamilton's modified principle.

(g) Show that if a generalized coordinate q_j is cyclic in L then it is cyclic in H.

(h) State the symplectic condition for a transformation to be canonical.

(i) Define Lagrange bracket of two quantities $u(q,p,t)$ and $v(q,p,t)$.

Q-3 (a) Using D'Alembert's principle show that $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j,\ j = 1, 2, \dots, n.$ [06]

(b) Obtain Lagrange's equations of motion for a particle moving in space under a force $\bar{F}$ using Cartesian coordinates. [06]

OR

(b) In usual notations, show that $T = T_0 + T_1 + T_2$. [06]

Q-4 (a) Derive the condition for extremum of the line integral $\int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$. [06]

(b) State and prove the law of conservation of angular momentum in Lagrangian formalism. [06]

OR

(b) Lagrangian of the spherical pendulum is given by [06]

$$L = \frac{m}{2} l^2(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + mgl\cos\theta.$$

Evaluate the energy function. Is it conserved? Justify.

Q-5 (a) Using an appropriate Legendre transformation derive Lagrange's equations of motion from Hamilton's equations of motion. [06]

(b) Discuss the principle of least action. [06]

OR

(b) Describe the Routhian procedure and state the Routhian equations of motion. [06]

Q-6 (a) Prove that an infinitesimal transformation is canonical. [06]

(b) Show that the transformation $Q = \log\left(\frac{\sin p}{q}\right)$, $P = q\cot p$ is canonical. [06]

OR

(b) If $\bar{F} = (F_1, F_2, F_3)$, $\bar{G} = (G_1, G_2, G_3)$ and $\bar{L}$ denotes angular momentum, then show that $[\bar{F}\cdot\bar{G}, \hat{n}\cdot\bar{L}] = 0$. [06]

——— × ———

2