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SEAT No._____

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Sardar Patel University

M.Sc. (Sem-I), PS01CMTH25, Methods of Differential Equations;
Thursday, 01st November, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The degree of differential equation $y'' + \sqrt{y'} + xy = 0$ is
   (A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 4
2. The set of ordinary points of $xy'' + (\sin x)y' + xy = 0$ is
   (A) $\mathbb{R} \setminus \{0\}$ (B) φ (C) $\{0\}$ (D) none of these
3. $\int_{-1}^{1} J_2(x)J_3(x)dx =$
   (A) $\sqrt{\pi}$ (B) -1 (C) 0 (D) none of these
4. $\int_0^1 x^2 P_4(x)dx =$
   (A) -1 (B) $\frac{1}{5}$ (C) $\frac{1}{8}$ (D) none of these
5. Which of the following is an integrating factor of $ydx + xdy$?
   (A) $\frac{1}{y^2}$ (B) $\frac{1}{x}$ (C) $\frac{1}{xy}$ (D) none of these
6. Which one is not homogeneous Pfaffian differential equation?
   (A) $xydx + yzdy + z^2dz = 0$ (B) $(x+y)dx + (y+z)dy + (z+x)dz = 0$
   (C) $x^2ydx + y^2zdy + z^2xdz = 0$ (D) none of these
7. $F(-2, \frac{1}{4}; \frac{1}{4}; 1) =$
   (A) -2 (B) 2 (C) 0 (D) none of these
8. The radius of convergence of Gauss's hypergeometric series is
   (A) 0 (B) 2 (C) 3 (D) 1

Q.2 Attempt any ***seven***: [14]

(a) Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{n+1}{2n+1}x^n$.
(b) Define regular singular point.
(c) Show that $\Gamma(x+1) = x\Gamma(x)$ where $x > 0$.
(d) Find $J_{\frac{1}{2}}(x)$.
(e) State orthogonality of Bessel's functions.
(f) State Picard's theorem.
(g) Find $F(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; x^2)$.
(h) Find a partial differential equation by eliminating F from $F(x+y, x-\sqrt{z}) = 0$.
(i) Define Pfaffian differential equation in n variables.

Q.3

(a) Solve: $y'' - xy' - y = 0$ near 0. [6]

(b) Solve: $2x^2y'' + 3xy' - (x+1)y = 0$ near 0. [6]

OR

(b) Solve: $x^2y'' + 5xy' + (4-x)y = 0$ near 0.

Q.4

(a) Prove: $\frac{d}{dx}[x^{-\alpha}J_\alpha(x)] = -x^{-\alpha}J_{\alpha+1}(x),\ \alpha \geq 0.$ [6]

(b) State and prove orthogonality of Legendre's polynomials. [6]

OR

(b) Express $x^4 - 3x^2 + x$ in terms of Legendre's polynomials.

Q.5

(a) Solve $y' = x + y,\ y(0) = 1$ using Picard's method of successive approximations. [6]

(b) State and prove integral representation of Gauss's hypergeometric function. [6]

OR

(b) Show that $P_n(x) = F\left(-n, n+1; 1; \frac{1-x}{2}\right)$.

Q.6

(a) Show that $X \cdot \text{curl} X = 0$ iff $\mu X \cdot \text{curl}(\mu X) = 0$ where $X = (P, Q, R)$ and $P, Q, R, \mu(\neq 0)$ are functions of x, y and z. [6]

(b) Solve: $z(xp - yq) = y^2 - x^2$. [6]

OR

(b) Verify that the differential equation $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$ is integrable and find its primitive.