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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination
Friday, 26th October, 2018
PS01CMTH22, Topology-I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) A countable intersection of open sets in ____ topology on $\mathbb{R}$ is open.
(i) cofinite (ii) usual (iii) cocountable (iv) lower limit

(b) For a topological space X and $A, B \subset X$, ____ $\neq$ ____.
(i) $\operatorname{bd} A$, $\operatorname{bd}(X \setminus A)$ (ii) $\overline{X \setminus A}$, $X \setminus A^\circ$ (iii) $\overline{A \cup B}$, $\overline{A} \cup \overline{B}$ (iv) $\overline{A \cap B}$, $\overline{A} \cap \overline{B}$

(c) Diameter of $\mathbb{N}$ as a subset of $\mathbb{R}$ with the metric $d(x,y) = \frac{|x-y|}{1+|x-y|}$, $(x, y \in \mathbb{R})$ is ____
(i) 1 (ii) 2 (iii) 3 (iv) ∞

(d) Projections need not be ____.
(i) open (ii) closed (iii) continuous (iv) onto

(e) In the ____ topology on $\mathbb{R}$, a connected subset must be singleton.
(i) indiscrete (ii) cofinite (iii) usual (iv) upper limit

(f) ____ $\subset \mathbb{R}$ is compact.
(i) $\{\pm\frac{1}{n}\} \cup \{0\}$ (ii) $\{7 \pm \frac{7}{n}\} \cup \{0\}$ (iii) $[-1, 1] \setminus \{0\}$ (iv) $\mathbb{Q}$

(g) Compact subset of ____ space is normal.
(i) connected (ii) T_2 (iii) T_1 (iv) cofinite

(h) A ____ space is normal.
(i) topological (ii) metric (iii) T_2 (iv) regular

Q-2 Attempt *Any Seven* of the following: [14]

(a) Show that the cocountable topology and the usual topology on $\mathbb{R}$ are not comparable.

(b) Show that $A \subset [0, 1]$ is closed in $[0, 1]$ if and only if A is closed in $\mathbb{R}$.

(c) Consider $\mathbb{R}$ with the cofinite topology. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f^{-1}(x)$ is closed in $\mathbb{R}$ for every $x \in \mathbb{R}$. Show that f is continuous.

(d) Give an example of a bounded subset of a metric space which is not closed. What is the diameter of the set given by you?

(e) Define *second countable space* and prove that $\mathbb{R}$ is second countable.

(f) Show that $\mathbb{R}$ with the discrete topology is disconnected.

(g) Define a T_4-space and show that a discrete space is T_4.

(h) State Urysohn's Lemma.

(i) Show that $\mathbb{R}$ is a T_1-space.

Q-3 (j) Define a *topological space* and a *basis for a topology*. Show that $\mathscr{T} = \{G \subset \mathbb{R} : G = \mathbb{R} \text{ or } G \cap \mathbb{Q} = \emptyset\}$ is a topology on $\mathbb{R}$. [6]

(k) Define a *limit point* of a subset of a topological space. Find all limit points of $\mathbb{Z} \subset \mathbb{R}$ in each of the cofinite topology and usual topology. [6]

OR

(k) Show, on $\mathbb{R}$, that the intersection of the lower limit topology and the upper limit topology is the usual topology. [6]

Q-4 (l) State and prove the pasting lemma. [6]

(m) Define *homeomorphism* and prove that homeomorphic image of a T_2-space is a T_2-space. [6]

OR

(m) Let X be a metric space, $x \in X$ and $A \subset X$. Show that $x \in \overline{A}$ if and only if there is a sequence $\{x_n\}$ in A such that $x_n \to x$. [6]

Q-5 (n) For $a, b \in \mathbb{R}$ with $a < b$, show that (a, b) is connected. [6]

(o) Let $(X, \mathscr{T})$ be a topological space and $(Y, \mathscr{T}_Y)$ be its subspace. Show that Y is compact in Y if and only if Y is compact in X. [6]

OR

(o) Show that a closed subset of a compact space is compact. [6]

Q-6 (p) Show that a topological space X is T_3 if and only if for every open set $G \subset X$ and a point $x \in G$, there exists an open set $H \subset X$ such that $x \in H \subset \overline{H} \subset G$. [6]

(q) Show that every metric space is T_4. [6]

OR

(q) State and prove Baire's Category Theorem. [6]

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