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Sc

SEAT NO.______

No. of printed pages 2

Sardar Patel University

M.Sc. - Mathematics (I SEM)

PS01CMTH21-Complex Analysis I

Time:10.00 a.m. to 01.00 p.m.

M.Sc.Ist Semester

Total Marks: 70

Date: 22-10-2018 Monday

Q.1 Choose the most appropriate option in the following questions. [08]

1. $Arg(1+i) + Arg(-1-i) =$ ______.

   (a) 0 (b) π (c) 2π (d) None of these

2. Suppose z is either real or purely imaginary. Then

   (a) $z^2 = \bar{z}$ (b) $(\bar{z})^2 = z$ (c) $(\bar{z})^2 = z^2$ (d) None of these

3. The set of singularity of the function $f(z) = \frac{1}{\sin\frac{\pi}{z}}$ is

   (a) $\{0\}$
   (b) $\{\frac{1}{n} : n \in \mathbb{Z}\setminus\{0\}\}$
   (c) $\{0, \frac{1}{n} : n \in \mathbb{Z}\setminus\{0\}\}$
   (d) None of these

4. If $f(z) = |z|^2$ for all $z \in \mathbb{C}$, then

   (a) f is not differentiable at 0
   (b) f is analytic at 0
   (c) f is entire function
   (d) None of these

5. If C is the unit circle taken in the positive direction, then $\int_C \frac{1}{\bar{z}}dz =$ ______.

   (a) $2\pi i$ (b) 0 (c) 1 (d) None of these

6. Let $f(z) = \dfrac{7z^6 + 5z^4 + 3z^2 + 1}{z^3}$. If C is the ellipse whose equation in $\mathbb{R}^2$ is $\frac{x^2}{4} + \frac{y^2}{9} = 1$ oriented counterclockwise, then $\int_C f(z)dz =$ ______.

   (a) πi (b) $2\pi i$ (c) 0 (d) None of these

7. $f(z) = \sin\frac{1}{z}$ has

   (a) no singularity in the plane
   (b) only a pole at 0
   (c) an essential singularity only at 0
   (d) None of these

8. Let T is a linear fraction transformation such that $T(0) = 0$, $T(1) = 1$, and $T(\infty) = \infty$. Then

   (a) T is a constant map
   (b) T must be identity map
   (c) no such T exists
   (d) None of these

Q.2 Attempt any seven. [14]

1. If $\lim_{z\to z_0} f(z) = w_0$ and $0 \neq w_0 \in \mathbb{C}$, then show that there is $c > 0$ and $\delta > 0$ such that $|f(z)| \geq c$ for all z satisfying $0 < |z - z_0| < \delta$.

2. If $z_1, z_2 \in \mathbb{C}$, then show that $\big||z_1| - |z_2|\big| \leq |z_1 - z_2|$.

3. Suppose that f is analytic in a domain D. If f is real valued, then show that f is a constant map.

(P.T.O.)

4. Is the mean value theorem true for complex valued functions of real variable? Justify your answer.

5. Is the converse of Cauchy-Goursat theorem true? Justify your answer.

6. State and prove Gauss's mean value theorem.

7. State Weierstrass M-Test.

8. If the origin is a fixed point of a linear fractional transformation, then prove that the transformation can be written in the form $w = \frac{z}{Cz+D}$, where $D \neq 0$.

9. Show that the transformation $w = \frac{i-z}{i+z}$ maps the the half plane $\mathrm{Im} z > 0$ onto the disc $|w| < 1$.

Q.3

(a) What are the necessary conditions for the existence of the derivative of a function at a point? Are they sufficient? Justify your answer. [06]

(b) Suppose that $f(z) = u(x,y) + iv(x,y)$; $z = x + iy$ and $z_0 = x_0 + iy_0$, $w_0 = u_0 + iv_0$. Show that $\lim_{z\to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$. [06]

OR

(b) Suppose that $\lim_{z\to z_0} f(z) = w_0 \in \mathbb{C}$. Show that (1) f is a bounded in a neighborhood of z_0. (2) For given $\epsilon > 0$ there is $\delta > 0$ such that $|f(z_1) - f(z_2)| < \epsilon$ whenever $0 < |z_1 - z_0| < \delta$ and $0 < |z_2 - z_0| < \delta$. [06]

Q.4

(a) Let $f = u + iv$ be analytic in a domain D. Show that both the functions u and v are harmonic in D. Is the converse true? Justify your answer. [06]

(b) Define integral of $w(t) = u(t) + iv(t)$ on $[a,b]$. State and prove Integral Inequality. [06]

OR

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the imaginary part
$v(x,y) = e^{2x}\sin 2y - y$. [06]

Q.5

(a) If a function f is analytic and not constant in a domain D, then show that $|f|$ has no maximum value in D. State the result you use. [06]

(b) State and prove Cauchy's theorem. State the results you use. [06]

OR

(b) State and prove Morera's theorem. State the results you use. [06]

Q.6

(a) State Cauchy Residue theorem. Use it to evaluate (1) $\int_{|z|=3} \frac{\exp(-z)}{z^2} dz$. (2) $\int_{|z|=3} \frac{\exp(-z)}{(z-1)^2} dz$. [06]

(b) Evaluate $\int_0^\infty \frac{\sin x}{x} dx$. [06]

OR

(b) Evaluate $\int_0^\infty \frac{\cos 3x}{x^2+1} dx$. [06]

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