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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination
Friday, 26th October, 2018
PS01CMTH02, Topology-I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) _____ is not a closed subset of $\mathbb{R}$.
(i) $[0,1]$ (ii) $\mathbb{Q}$ (iii) $\mathbb{N}$ (iv) $\{\frac{1}{x} : x > 0\} \cup \{0\}$

(b) For a topological space X and $A, B \subset X$, _____ $\neq$ _____.
(i) $\operatorname{bd} A, \operatorname{bd}(X \setminus A)$ (ii) $\overline{X \setminus A}, X \setminus A^{\circ}$ (iii) $\overline{A \cup B}, \overline{A} \cup \overline{B}$ (iv) $\overline{A \cap B}, \overline{A} \cap \overline{B}$

(c) $\mathbb{R} \times \mathbb{R}$ is not _____.
(i) compact (ii) complete (iii) T_1 (iv) regular

(d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) =$_____, $(x \in \mathbb{R})$, is not uniformly continuous.
(i) $\sin x$ (ii) x^3 (iii) $\sin^2 x + 2\cos 2x$ (iv) x

(e) In the _____ topology on $\mathbb{R}$, a connected subset must be singleton.
(i) indiscrete (ii) cofinite (iii) usual (iv) upper limit

(f) _____ $\subset \mathbb{R}$ is compact.
(i) $\{\pm\frac{1}{n}\} \cup \{0\}$ (ii) $\{7 \pm \frac{7}{n}\} \cup \{0\}$ (iii) $[-1,1] \setminus \{0\}$ (iv) $\mathbb{Q}$

(g) Compact subset of _____ space is normal.
(i) connected (ii) T_2 (iii) T_1 (iv) cofinite

(h) A _____ space is normal.
(i) topological (ii) metric (iii) T_2 (iv) regular

Q-2 Attempt *Any Seven* of the following: [14]

(a) Give an example of a nonempty proper subset of $\mathbb{Q}$ which is closed as well as open.

(b) Show that $A \subset (0,1)$ is open in $(0,1)$ if and only if A is open in $\mathbb{R}$.

(c) Give a base for the product of finitely many topological spaces.

(d) Give an example of a bounded subset of a metric space which is not closed. What is the diameter of the set given by you?

(e) State Cantor's intersection theorem

(f) Show that a finite set with any topology is compact.

(g) Show that $\mathbb{R}$ with the discrete topology is not compact.

(h) Define a T_4-*space* and show that a discrete space is T_4.

(i) Show that $\mathbb{R}$ is a T_1-space.

(PTO)

Q-3 (j) Define a *topological space* and a *basis for a topology*. Show that $\mathscr{T} = \{G \subset \mathbb{R} : G = \mathbb{R} \text{ or } G \cap \mathbb{Q} = \emptyset\}$ is a topology on $\mathbb{R}$. [6]

(k) Define a *limit point* of a subset of a topological space. Find all limit points of $\mathbb{Z} \subset \mathbb{R}$ in each of the cofinite topology and usual topology. [6]

OR

(k) Show, on $\mathbb{R}$, that the intersection of the lower limit topology and the upper limit topology is the usual topology. [6]

Q-4 (l) Show that projections are continuous and open. [6]

(m) For a product space $X = \prod_{i=1}^{n} X_i$, show that X is T_1 if and only if each X_i is T_1. [6]

OR

(m) Show that in a metric space, every convergent sequence is Cauchy. Also show that a uniformly continuous function maps a Cauchy sequence to Cauchy sequence. [6]

Q-5 (n) Show that $\mathbb{R}$ with cofinite topology is compact. [6]

(o) Let $(X, \mathscr{T})$ be a topological space and $(Y, \mathscr{T}_Y)$ be its subspace. Show that Y is compact in Y if and only if Y is compact in X. [6]

OR

(o) Show that a closed subset of a compact space is compact. [6]

Q-6 (p) Show that a topological space X is T_4 if and only if for every open set $G \subset X$ and a closed set $E \subset G$, there exists an open set $H \subset X$ such that $E \subset H \subset \overline{H} \subset G$. [6]

(q) For $a, b \in \mathbb{R}$ with $a < b$, show that (a, b) is connected. [6]

OR

(q) Show that a compact subset of a T_2-space is closed. [6]

—X—