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SEAT NO.______

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No. of printed pages 2

Sardar Patel University

M.Sc.- Mathematics - Sem-I

PS01CMTH01-Complex Analysis I

Time:10.00 a.m. to 01.00 p.m. | Total Marks: 70

M.Sc.I^{st} Semester | Date: 22-10-2018 Monday

Q.1 Choose the most appropriate option in the following questions. [08]

1. The equation $|z-1| = |z+i|$ represents a

   (a) circle (b) ellipse (c) hyperbola (d) None of these

2. If C is any n^{th} root of unity other than unity, then $1+C+C^2+\cdots+C^{n-1} =$ ______.

   (a) 2^n (b) n (c) 0 (d) None of these

3. Which of the following is not a harmonic function ?

   (a) $u(x,y) = \dfrac{y}{x^2+y^2}$

   (b) $u(x,y) = x^2 - y^2$

   (c) $u(x,y) = e^{2018x}$

   (d) None of these

4. Which of the following are Cauchy-Riemann Equations in Polar Coordinates?

   (a) $ru_\theta = -v_r$, $rv_\theta = u_r$

   (b) $ru_r = -v_\theta$, $rv_r = u_\theta$

   (c) $ru_\theta = v_r$, $rv_\theta = -u_r$

   (d) None of these

5. Which of the following is a bounded function on $\mathbb{C}$?

   (a) $\cos z$ (b) e^z (c) z (d) none of these

6. If C is the unit circle taken in the positive direction, then $\int_C \frac{1}{\bar{z}} dz =$ ______.

   (a) $2\pi i$ (b) 0 (c) 1 (d) None of these

7. The set of singularity of the function $f(z) = \frac{1}{\sin\frac{\pi}{z}}$ is

   (a) $\{0\}$

   (b) $\{\frac{1}{n} : n \in \mathbb{Z}\backslash\{0\}\}$

   (c) $\{0, \frac{1}{n} : n \in \mathbb{Z}\backslash\{0\}\}$

   (d) None of these

8. $\int_{|z|=3} \frac{\exp(-z)}{z^2} dz =$ ______.

   (a) $2\pi i$ (b) $-2\pi i$ (c) 0 (d) None of these

Q.2 Attempt any seven. [14]

1. Find the locus of $|z-4i| + |z+4i| = 10$.
2. Find out the value of arg z+arg $\bar{z}$ for a nonzero complex number z.
3. When is $z_0 \in \mathbb{C}$ called a singularity of f? Determine the singularities of $\frac{1}{z}$.
4. Define entire function with example.

(P.T.O.)

5. Give any two anti-derivatives of $\cos z$.

6. State Gauss's mean value theorem.

7. Define simple closed contor with examlple.

8. Define mobius transformation.

9. Find the Laurent's series of $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$.

Q.3

(a) Suppose $w \in \mathbb{C}$ and $n \in \mathbb{N}$. Find all complex numbera z such that $z^n = w$. [06]

(b) State and prove triangle inequality. [06]

OR

(b) If z_1 and z_2 are nonzero complex numbers, then show that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. [06]

Q.4

(a) Obtain the necessary condition for the existence of derivative of a function at a point. [06]

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the imaginary part $v(x,y) = e^{2x}\sin 2y - y$. [06]

OR

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the real part $u(x,y) = y^3 - 3x^2y$. [06]

Q.5

(a) State and prove fundamental theorem of algebra. [06]

(b) Suppose that $|f(z)| \le |f(z_0)|$ at each point z in some neighborhood $|z - z_0| < \epsilon$ in which f is analytic. Show that f has the constant value $f(z_0)$ throughout that neighborhood. [06]

OR

(b) Let f be an entire function. If the real part of f is bounded above, then show that f is a constant map. [06]

Q.6

(a) State and prove Taylor's Theorem. [06]

(b) Evaluate $\int_0^\infty \frac{\sin x}{x} dx$. [06]

OR

(b) Evaluate $\int_0^\infty \frac{2x^2-1}{x^4+5x^2+4} dx$. [06]

—X—