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SARDAR PATEL UNIVERSITY
M.Sc. (Semester-I) Examination
October-2016
Friday 28/10/2016
Time: 10:00 AM to 01:00 PM

Subject: Mathematics **Course No.PS01EMTH02**

Mathematical Classical Mechanics

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) For a simple pendulum, degrees of freedom = _________.
(a) 0 (b) 1 (c) 3 (d) can not be determined

(2) The motion of a particle in the space is __________ constraint.
(a) not a (b) a holonomic (c) a non-holonomic (d) conservative

(3) The condition for extremum of $J = \int_{x_1}^{x_2} f(\dot{y}, x)dx$ is _______.
(a) $\frac{\partial f}{\partial \dot{y}} = const.$ (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial x} = 0$
(c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial \dot{y}} = 0$ (d) $\frac{d}{dy}\left(\frac{\partial f}{\partial \dot{x}}\right) - \frac{\partial f}{\partial y} = 0$

(4) If the Lagrangian L does not depend on t explicitly then __________ is conserved.
(a) all coordinates (b) energy function (c) all momenta (d) nothing

(5) Which one of the following is incorrect?
(a) $\frac{\partial H}{\partial p_j} = -q_j$ (b) $H = h$ (c) $\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$ (d) none of these

(6) If all coordinates are non-cyclic then Routhian $R =$ _______.
(a) H (b) L (c) $-H$ (d) $-L$

(7) Pick up the correct statement.
(a) Canonical transformations form a group.
(b) Jacobian matrix for a canonical transformation is singular.
(c) Inverse of a canonical transformation may not be a canonical transformation.
(d) None of the above is true.

(8) $[q_1, q_2] =$_________
(a) 0 (b) 1 (c) zero matrix (d) not defined

Q-2 Answer any **Seven.** (14)

(1) Define a holonomic constraint and give an example of it.
(2) State Lagrange's equations of motion in general form.
(3) Define configuration space.
(4) Define cyclic coordinate.
(5) State Hamilton's equations of motion in matrix form.
(6) State Hamilton's modified principle.
(7) State the transformation generated by a function of type F_2.
(8) Define Lagrange bracket.
(9) Show that $[p_y, L_z] = p_x$, notations being usual.

Q-3

(a) State Lagrange's equations of motion in general form and hence derive the form in case of conservative force. (06)

(b) A particle is moving inside the unit circle. Express the constraints in this case in mathematical form and hence classify them. (06)

OR

(b) Obtain Lagrange's equations of motion for a particle moving in a plane using polar coordinates.

Q-4

(a) State the condition for the extremum of the line integral (06)

$$J = \int_{x_1}^{x_2} f(y_1, y_2, \cdots, y_n, \dot{y}_1, \dot{y}_2, \cdots, \dot{y}_n, x)\,dx.$$

Hence derive Lagrange's equations of motion.

(b) Prove the law of conservation of linear momentum using Lagrangian formalism. (06)

OR

(b) Using calculus of variations obtain the curve for minimum surface of revolution.

Q-5

(a) State Legendre transformation and using it derive Lagrange's equations of motion from Hamilton's equations of motion. (06)

(b) Giving an example describe Routhian procedure. (06)

OR

(b) Lagrangian for a system of three degrees of freedom is given by $L = \frac{I_1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 - Mgl\cos\theta$. Obtain Hamiltonian.

Q-6

(a) Let u and v be two constants of motion, show that $[u, v]$ is also a constant of motion. (06)

(b) Show that the transformation, (06)

$Q = \log\left(1 + \sqrt{q}\cos p\right), P = 2\sqrt{q}\left(1 + \sqrt{q}\cos p\right)\sin p,$

is canonical.

OR

(b) Hamiltonian for a motion in one dimension with constant acceleration a is given by $H = \frac{p^2}{2m} - max$. Using Poisson bracket formalism obtain the expression for x subject to the conditions $x = x_0, p = p_0$ at $t = 0$.

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