[32] Seat No.: ________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester I) Examination

Date: 28-10-2016 **Time: 10.00 To 01.00**

Subject: MATHEMATICS **Paper No. PS01EMTH01 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) If $K_{1,n} = K_{n+1}$, then

(a) n = 1 (b) n = 2 (c) n > 2 (d) none of these

(2) A symmetric digraph is

(a) Euler (b) connected (c) regular (d) balanced

(3) For G = C_n with clockwise direction, rank(B) is

(a) n (b) n – 1 (c) 1 (d) none of these

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$

(a) ne (b) e (c) 2e (d) e^2

(5) The coefficient c_4 in chromatic polynomial of K_4 is

(a) 0 (b) 1 (c) 4 (d) 4!

(6) Which of the following graphs is not Hamiltonian?

(a) K_n (b) $K_{n,n}$ (c) P_n (d) C_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum ⇒ perfect (c) maximal ⇒ maximum

(b) maximum ⇒ maximal (d) maximum ⇒ perfect

(8) If G = P_{51}, then

(a) $\alpha(G) > \beta(G)$ (b) $\alpha(G) < \beta(G)$ (c) $\alpha(G) = \beta(G)$ (d) $|\alpha(G)| = |\beta(G)|$

2. Attempt any SEVEN: [14]

(a) Find the radius of $K_{m,n}$ (m, n ≥ 2).

(b) Prove or disprove: An Euler digraph is regular.

(c) Define incidence matrix in a digraph.

(d) Give an example of a spanning in tree which is also a spanning out tree in a digraph.

(e) Prove: If G = C_n and n is odd, then $\chi(G) = 3$.

(f) What is Four color problem?

(g) Why P_4 is not isomorphic to $K_{1,3}$?

(h) Prove: If $S \subset V(G)$ is a vertex cover, then V(G) – S is an independent set, in G.

(i) Prove or disprove: The graph K_{2n} has a perfect matching.

3. (a) Define the following with examples: [6]
(i) Asymmetric digraph (ii) Symmetric digraph (iii) Strongly connected digraph

(b) Prove: An arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

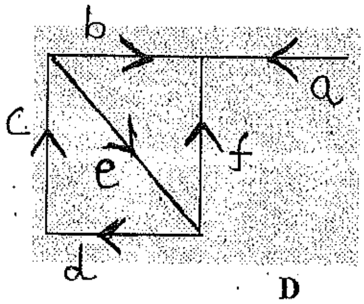
(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Let G be a connected digraph with n vertices. Prove that rank of $A(G) = n - 1$. [6]

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \dots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \dots n$. [6]

OR

(b) Define fundamental circuit matrix in a digraph and find it w. r. t. spanning tree $T = \{a, b, d, e\}$ in digraph D below: [6]



D

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Let G be a k-chromatic graph with n vertices. Prove that $n \leq k\,\alpha(G)$. [6]

OR

(b) Find the coefficients c_2 and c_3 of Chromatic polynomial of graph $K_{1,3}$. [6]

6. (a) Let G be a graph (no isolated vertex) with n vertices. Prove that $\alpha'(G) + \beta'(G) = n$. [6]

(b) State Hall's theorem & show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G)$, $\beta(G)$ and find it with the corresponding sets for $G = K_{n,m}$. [6]

x-x-x-x-x-x