(A-2)

Seat No.____

No of printed pages: 2

Sardar Patel University

M.Sc. Semester I Examination

2016

Friday, 21 October

10.00 am to 13.00 PM

Mathematics: PS01CMTH07

(Topology I)

Maximum Marks: 70

Q.1 Write the question number and correct **option number only** for each question. [8]

(*a*) $\{(a,b) : a, b \in \mathbb{Z}, a < b\}$ is _____.

(i) a base for a topology on $\mathbb{Q}$ (ii) a base for a topology on $\mathbb{N}$ (iii) an open cover of $\mathbb{R}$ (iv) family of disjoint intervals

(*b*) All topologies on _____ are compact.

(i) a finite set (ii) an infinite set (iii) $\mathbb{Q}$ (iv) $\mathbb{N}$

(*c*) A compact metric space must be _____.

(i) countable (ii) finite (iii) complete (iv) infinite

(*d*) $\mathbb{R}$ with _____ topology is T_2 and separable.

(i) cocountable (ii) discrete (iii) usual (iv) cofinite

(*e*) _____ with usual metric is not complete.

(i) $\mathbb{Z}$ (ii) $\mathbb{Q}$ (iii) $[0,1]$ (iv) $\mathbb{R}$

(*f*) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = -x$ is discontinuous if $\mathbb{R}$ has _____ topology.

(i) discrete (ii) cocountable (iii) cofinite (iv) lower limit

(*g*) Projections are _____.

(i) closed (ii) open (iii) one-one (iv) homeomorphism

(*h*) A compact T_2-space is

(i) discrete (ii) T_3 (iii) separable (iv) bounded

Q.2 Attempt any **Seven. (Start a new page.)** [14]

(*a*) Prove that $\{\{-n, n\} : n \in \mathbb{Z}\}$ is a base for some topology on $\mathbb{Z}$.

(*b*) Define *interior of a subset* of a topological space. Find the interior of $[0,1]$ in $\mathbb{R}$ with the usual topology.

(*c*) Define *closure of a subset* of a topological space. Find closure of $(0,1)$ in $\mathbb{R}$

(*d*) Show that product of indiscrete topological spaces is an indiscrete topological space.

(*e*) Show that a constant function is always continuous.

(*f*) Show that $\{(0,n) : n \in \mathbb{N}\}$ has finite intersection property.

(*g*) Define a *bounded* subset of a metric space and show that $\mathbb{Q}$ is not bounded.

(*h*) Define T_1-*space* and show that $\mathbb{R}$ is T_1.

(*i*) In usual notations prove that $\overline{(X \setminus A)} = X \setminus A^\circ$.

(P.T.O.)

[Contd...]

Q.3 **(Start a new page.)**

(*a*) Show that intersection of two topologies on $\mathbb{R}$ is again a topology on $\mathbb{R}$, but union of two topologies on $\mathbb{R}$ need not be a topology on $\mathbb{R}$. [6]

(*b*) Show that arbitrary union of closed sets need not be closed. Is a finite union of closed sets is closed? Justify. [6]

OR

(*b*) Show that $A \subset \mathbb{R}$ is closed if and only if it contains its boundary. [6]

Q.4 **(Start a new page.)**

(a) Define a *continuous function* and show that composition of two continuous functions is a continuous function. [6]

(b) Let X be a complete metric space and $\{F_n : n \in \mathbb{N}\}$ be a family of closed subsets of X such that $F_{n+1} \subset F_n$ for all $n \in \mathbb{N}$. If $\mathrm{diam}(F_n) \to 0$, then show that $\bigcap_{n=1}^{\infty} F_n$ is singleton. [6]

OR

(b) Show that projections are continuous and open. [6]

Q.5 **(Start a new page.)**

(a) Define a *compact* topological space and show that a closed subspace of a compact space is compact. [6]

(b) Show that a sequentially compact metric space has Bolzano-Weierstrass Property. [6]

OR

(b) Show that continuous image of a compact space is compact. [6]

Q.6 **(Start a new page.)**

(a) Define a T_3-*space* and show that a subspace of a T_3-space is T_3. [6]

(b) Show that a metric space is T_4. [6]

OR

(b) Define a T_4-*space* and show that a normal space is T_3. [6]

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2