Seat No.:

[35]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH05, Methods of Differential Equations;
Wednesday, 26^{th} October, 2016; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The degree of differential equation $x^4y'' - (y'+x)^{\frac{1}{2}} = 0$ is
   (A) 1 (B) $\frac{1}{2}$ (C) 2 (D) 4
2. The set of ordinary points of $xy'' + xy' + (1-e^x)y = 0$ is
   (A) $\{0\}$ (B) φ (C) $\{1\}$ (D) none of these
3. $(2\Gamma(\frac{3}{2})) =$
   (A) $\sqrt{\pi}$ (B) $\sqrt{\frac{\pi}{2}}$ (C) $2\sqrt{\pi}$ (D) none of these
4. $\int_0^1 (3x^2-1)P_2(x)dx =$
   (A) 0 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) none of these
5. Which one is not an integrating factor of $yzdx + xzdy + xydz$?
   (A) $\frac{1}{xyz}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{x^2}$
6. The differential equation obtained from $z = (x-a)^2 + (y-b)^2$ by eliminating a, b, is
   (A) $p^2+q^2 = 4z$ (B) $p+q = 4z$ (C) $p^2+q^2 = z$ (D) none of these
7. $F(\alpha, \beta; \gamma; -1)$ converges if
   (A) $\gamma < \alpha+\beta-1$ (B) $\gamma < \alpha+\beta+1$
   (C) $\gamma > \alpha+\beta-1$ (D) none of these
8. $\frac{d}{dx}[F(\alpha,\beta;\gamma;x)]_{x=0}$ equals
   (A) 1 (B) 0 (C) -1 (D) $\frac{\alpha\beta}{\gamma}$

Q.2 Attempt any ***seven***: [14]

(a) Find the radius of convergence of $\sum\limits_{n=1}^{\infty} \frac{n!}{n^n} x^{2n}$.
(b) State Frobenius theorem.
(c) Verify that $y(x) = e^x$ is a solution of $xy'' - (2x+1)y' + (x+1)y = 0$ and hence find the general solution.
(d) Prove: $\Gamma(x) = (x-1)\Gamma(x-1), x > 1$.
(e) Show that between any two positive roots of J_1 there is a root of J_0.
(f) Find a partial differential equation by eliminating a, b and c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
(g) Solve: $x^3p + y^3q = z^3$.
(h) Find $F(1, \frac{1}{2}; \frac{1}{2}; \frac{1}{2})$.
(i) Solve: $z = px + qy + p^2 + q^2$.

Q.3

(a) Solve: $y'' - xy = 0$ near 0. [6]

(b) Solve: $2x^2y'' + 3xy' - (1+x)y = 0$ near 0. [6]

OR

(b) Prove or disprove: the function $f : (-1, 1) \to \mathbb{R},\ f(x) = \frac{1}{1-x}$ is analytic at 0.

Q.4

(a) Prove: $\frac{d}{dx}[x^{-\alpha}J_\alpha(x)] = -x^{-\alpha}J_{\alpha+1}(x),\ \alpha \geq 0.$ [6]

(b) State and prove orthogonality of Legendre's polynomials. [6]

OR

(b) Let $p(x)$ be a polynomial of degree $n \geq 1$ with

$$\int_{-1}^{1} x^k p(x)dx = 0 \quad \text{where} \quad k = 0, 1, 2, ..., n-1,$$

show that $p(x) = cP_n(x)$ for some constant c, where P_n is a Legendre polynomial.

Q.5

(a) Find a necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$, a relation $F(u, v) = 0$ not involving x or y explicitly. [6]

(b) Solve $y' - x - y = 0,\ y(0) = 1$ using Picard's method of successive approximations. [6]

OR

(b) Verify that the differential equation $x(y^2 - 1)dx + y(x^2 - z^2)dy - z(y^2 - 1)dz = 0$ is integrable and find its primitive.

Q.6

(a) Show that $P_n(x) = F\big(-n, n+1; 1; \frac{1-x}{2}\big)$. [6]

(b) Solve: $z^2 = pqxy$ using Charpit's method. [6]

OR

(b) Solve: $p^2x + q^2y = z$ using Jacobi's method.

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